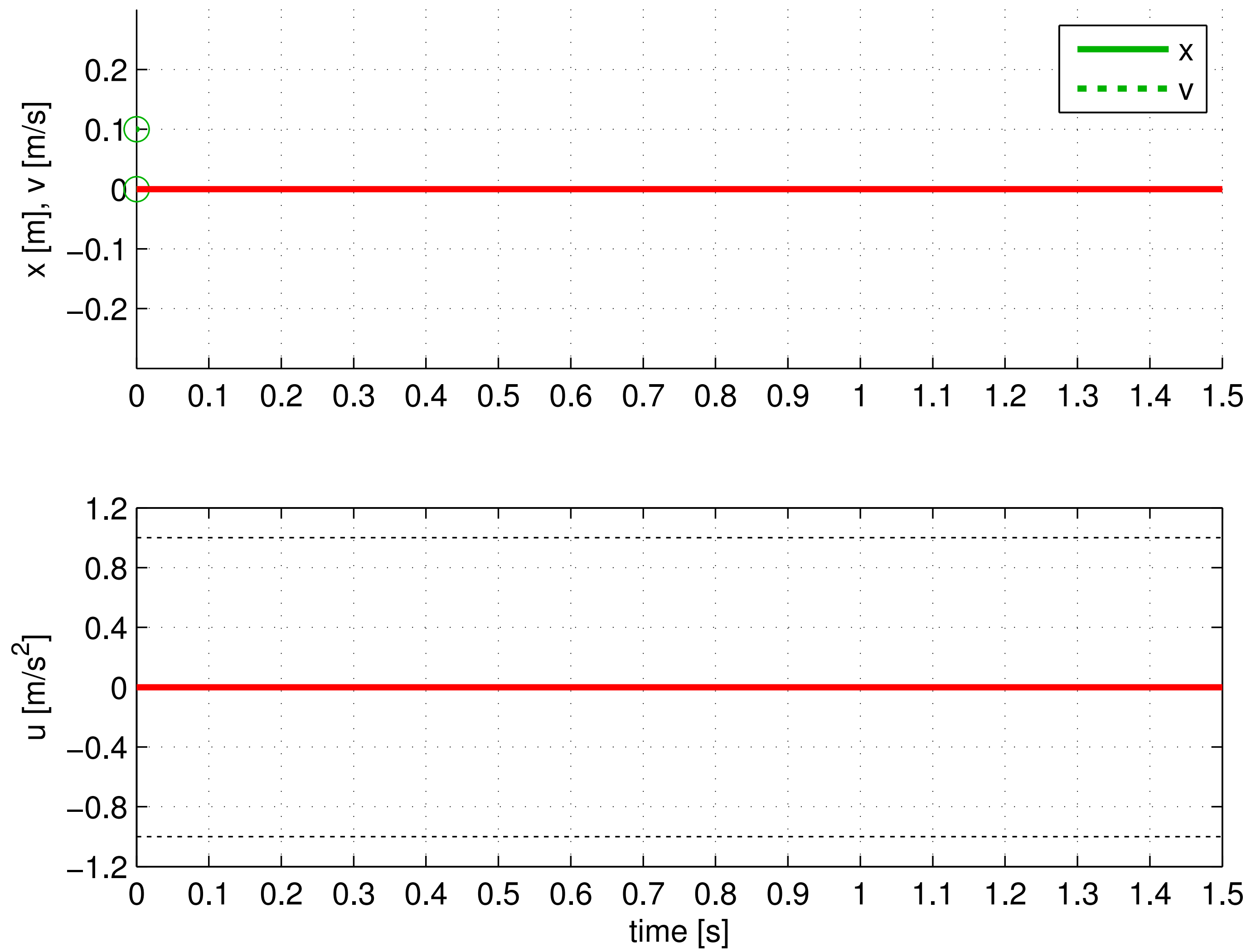
Fast solvers for nonlinear optimal control and estimation with applications to tethered kites

Milan Vukov, Rien Quirynen and Moritz Diehl

KU LEUVEN

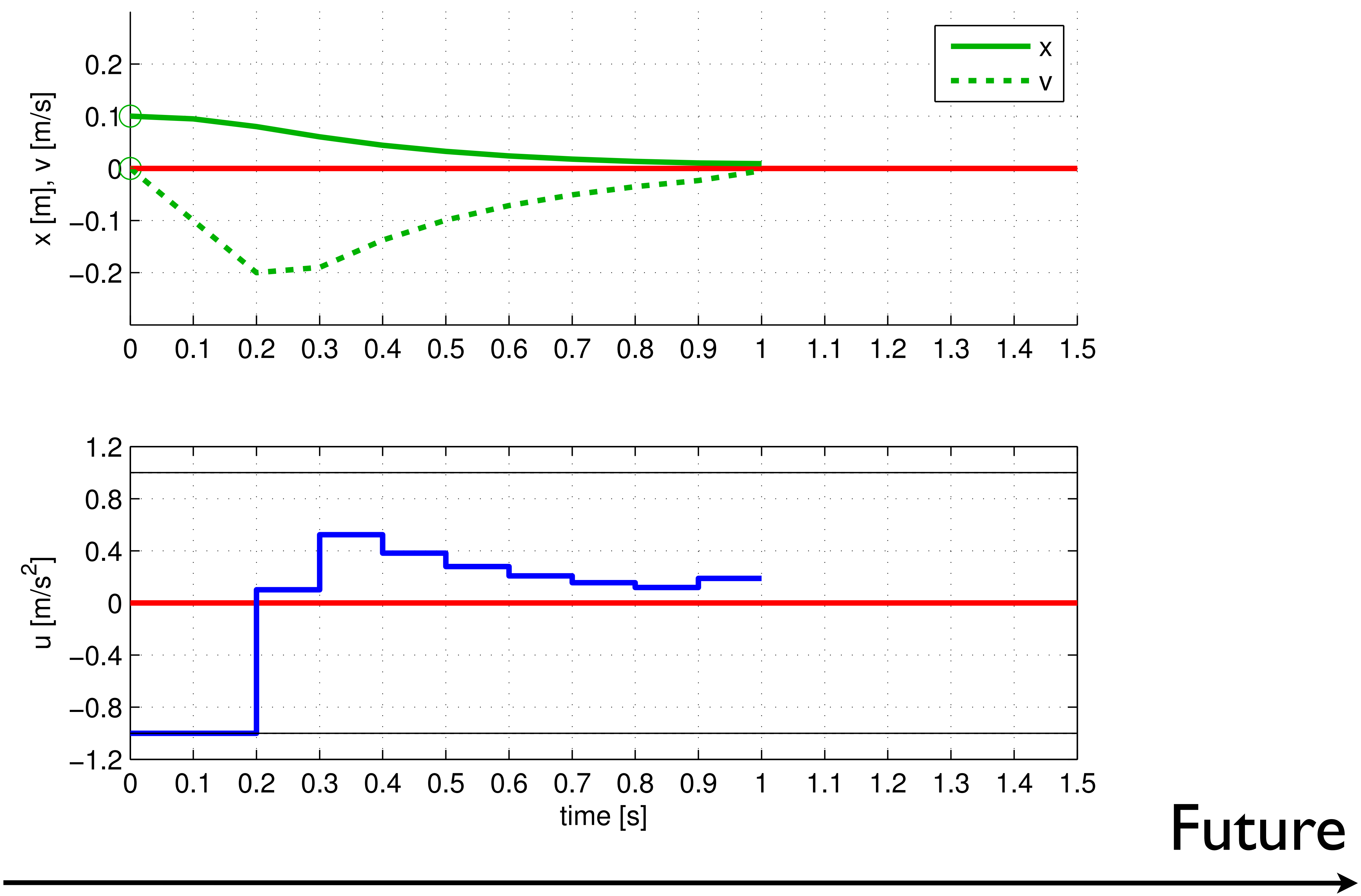
ERC HIGHWIND

Nonlinear Model Predictive Control

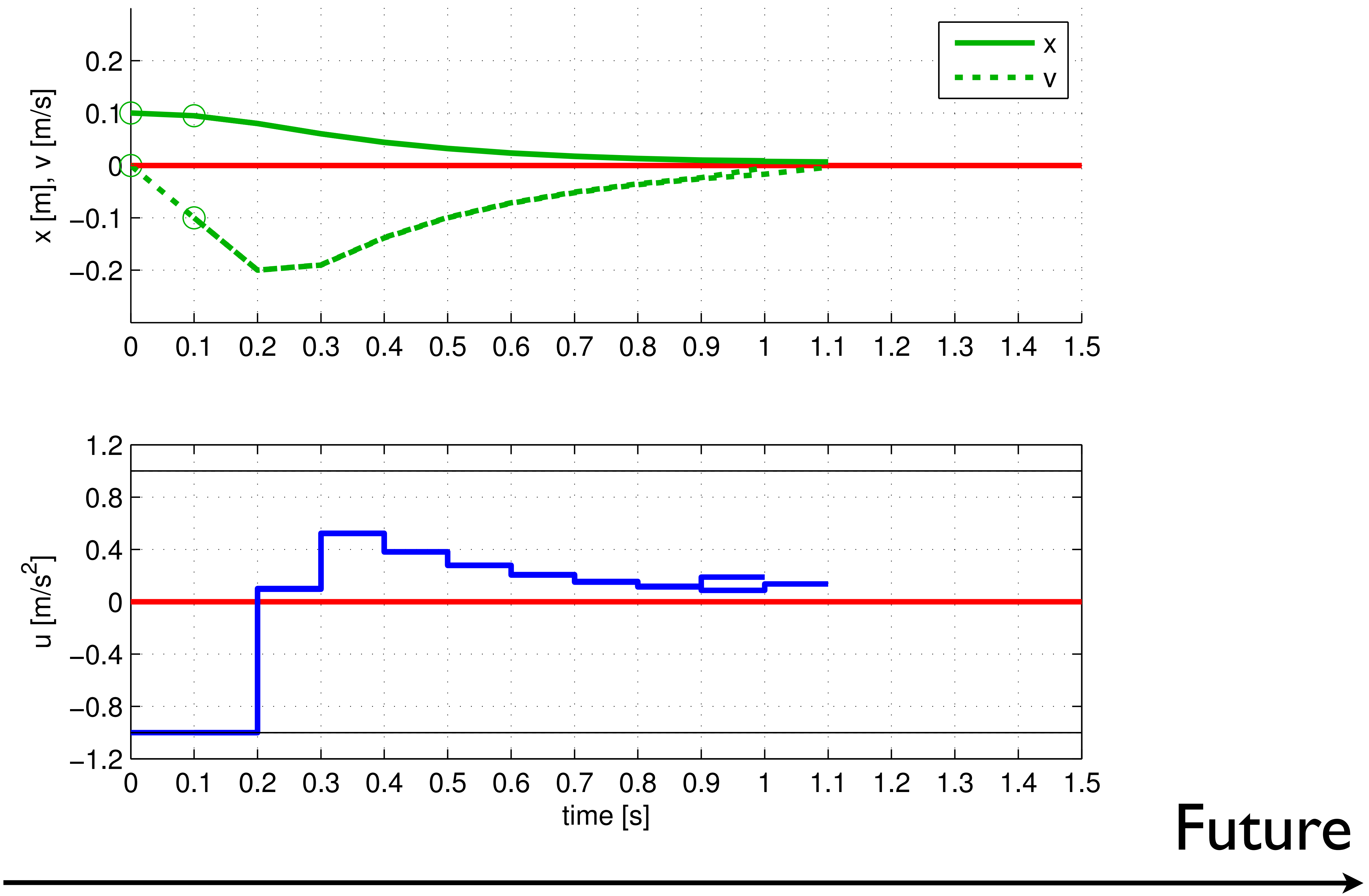


$$
\begin{aligned}
\min_{u,s} \quad & \|s_P - s_{ref}\|^2_{Q_P} + \sum_{k=0}^{P-1} \|s_k - s_{ref}\|^2_Q + \|u_k - u_{ref}\|^2_R && \rightarrow \textbf{deviation} \text{ from the reference} \\
\text{s.t.} \quad & s_{k+1} = f(s_k, u_k), \quad k = 0, \dots, P-1, && \rightarrow \textbf{model} \text{ of the system evolution} \\
& h(s_k, u_k) \leq 0, \quad k = 0, \dots, P-1, && \rightarrow \textbf{constraints} \\
& s_0 = \hat{x}_0 && \rightarrow \textbf{current state} \text{ of the system}
\end{aligned}
$$

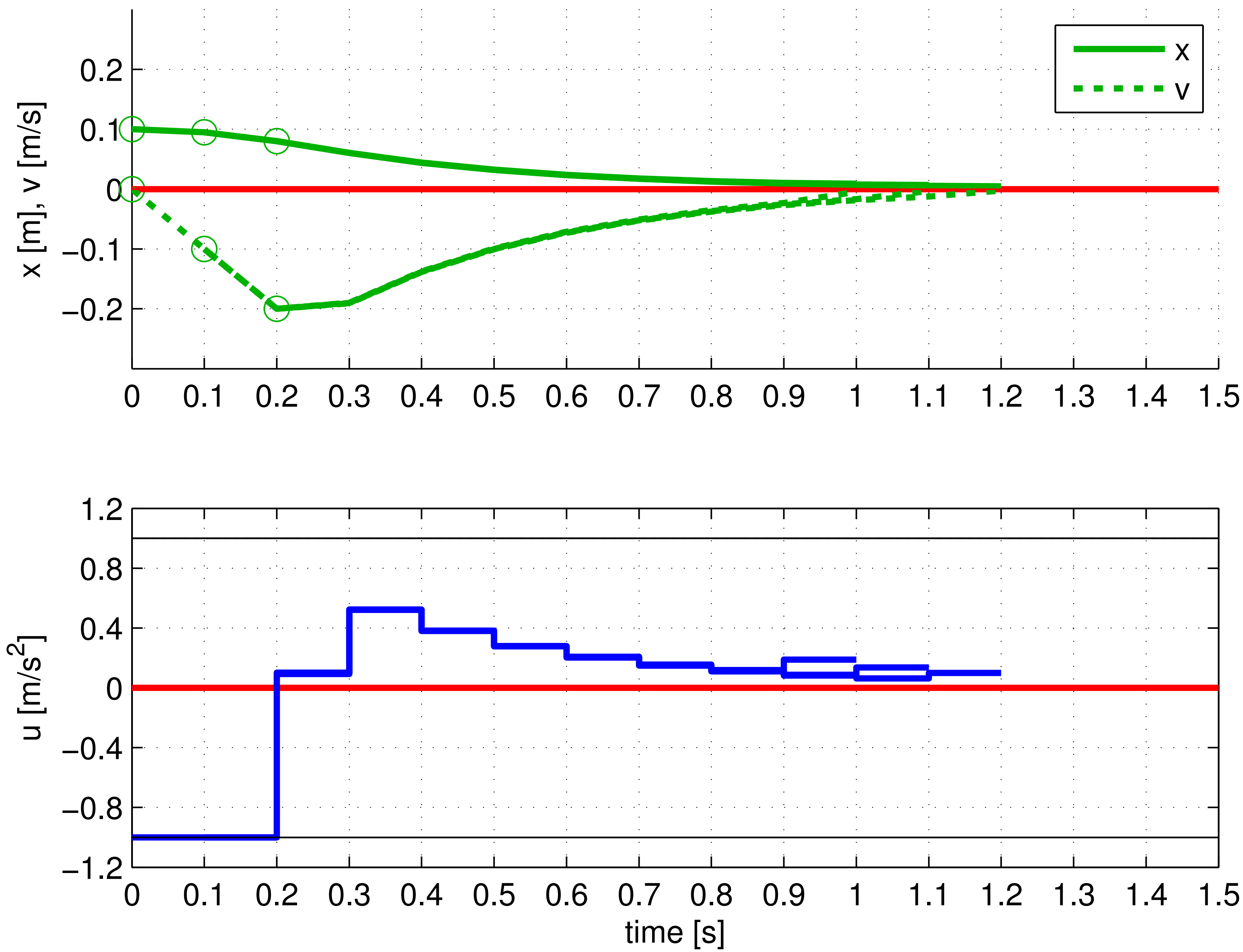
Nonlinear Model Predictive Control



Nonlinear Model Predictive Control

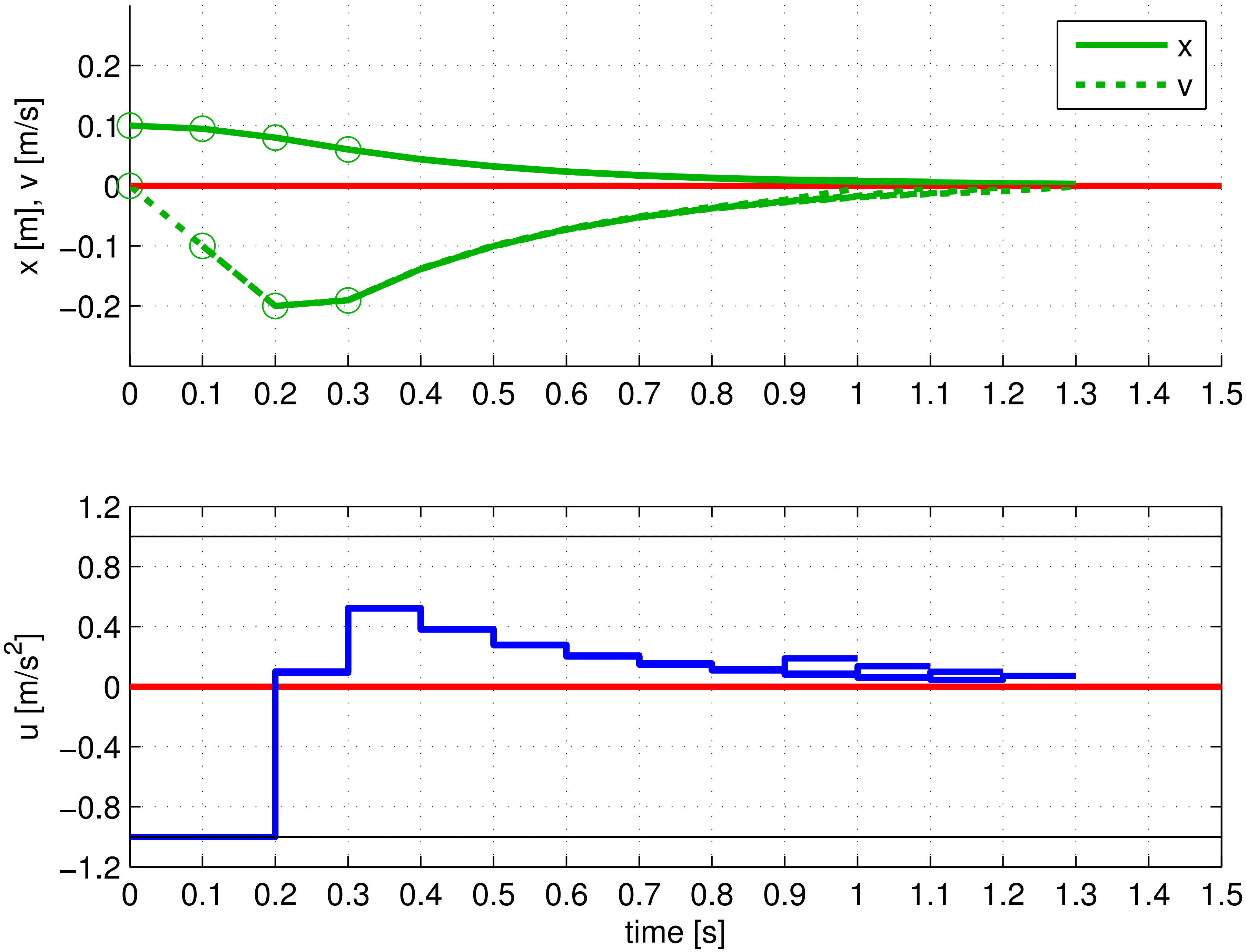


Nonlinear Model Predictive Control



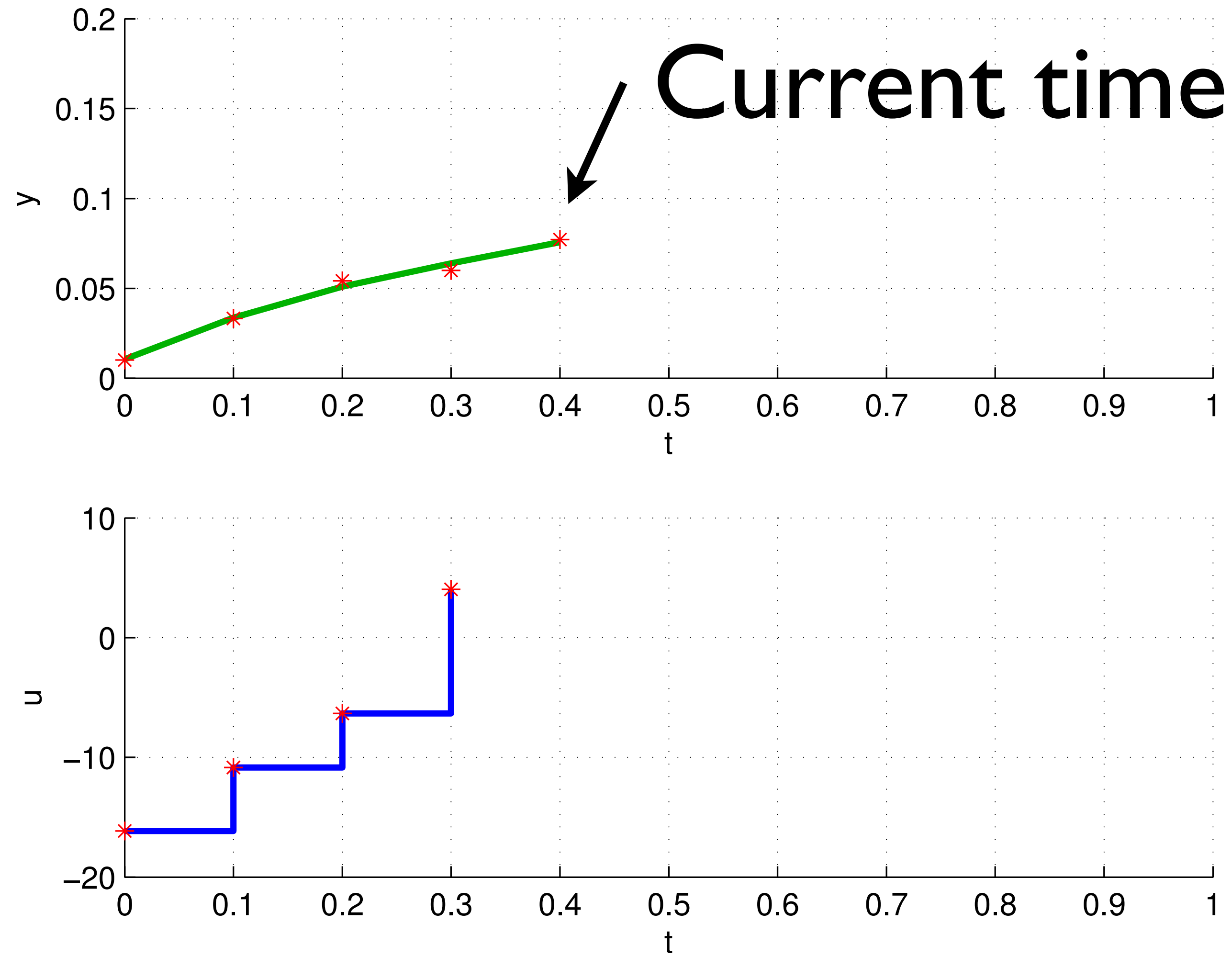
Future

Nonlinear Model Predictive Control



Future

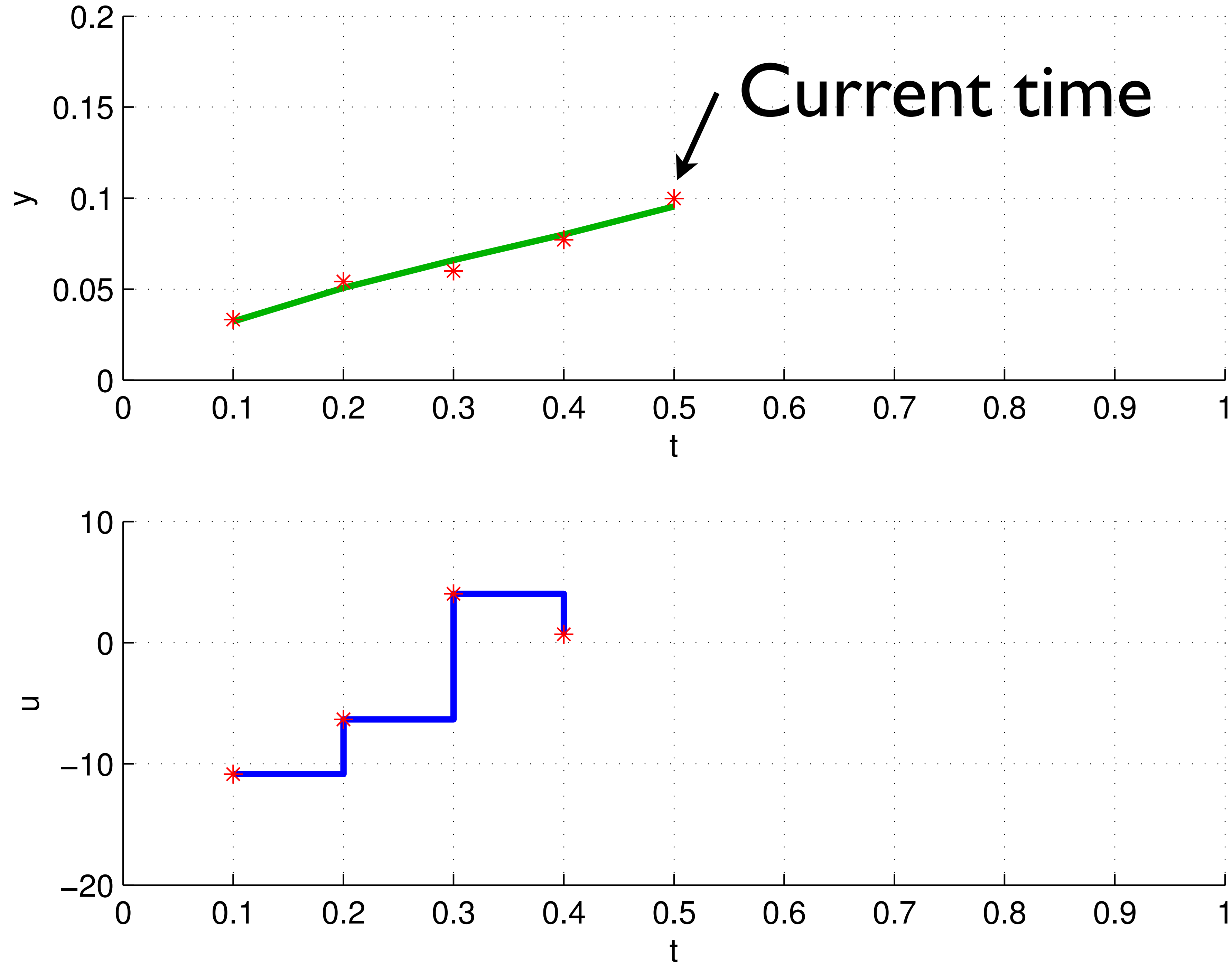
Nonlinear Moving Horizon Estimation



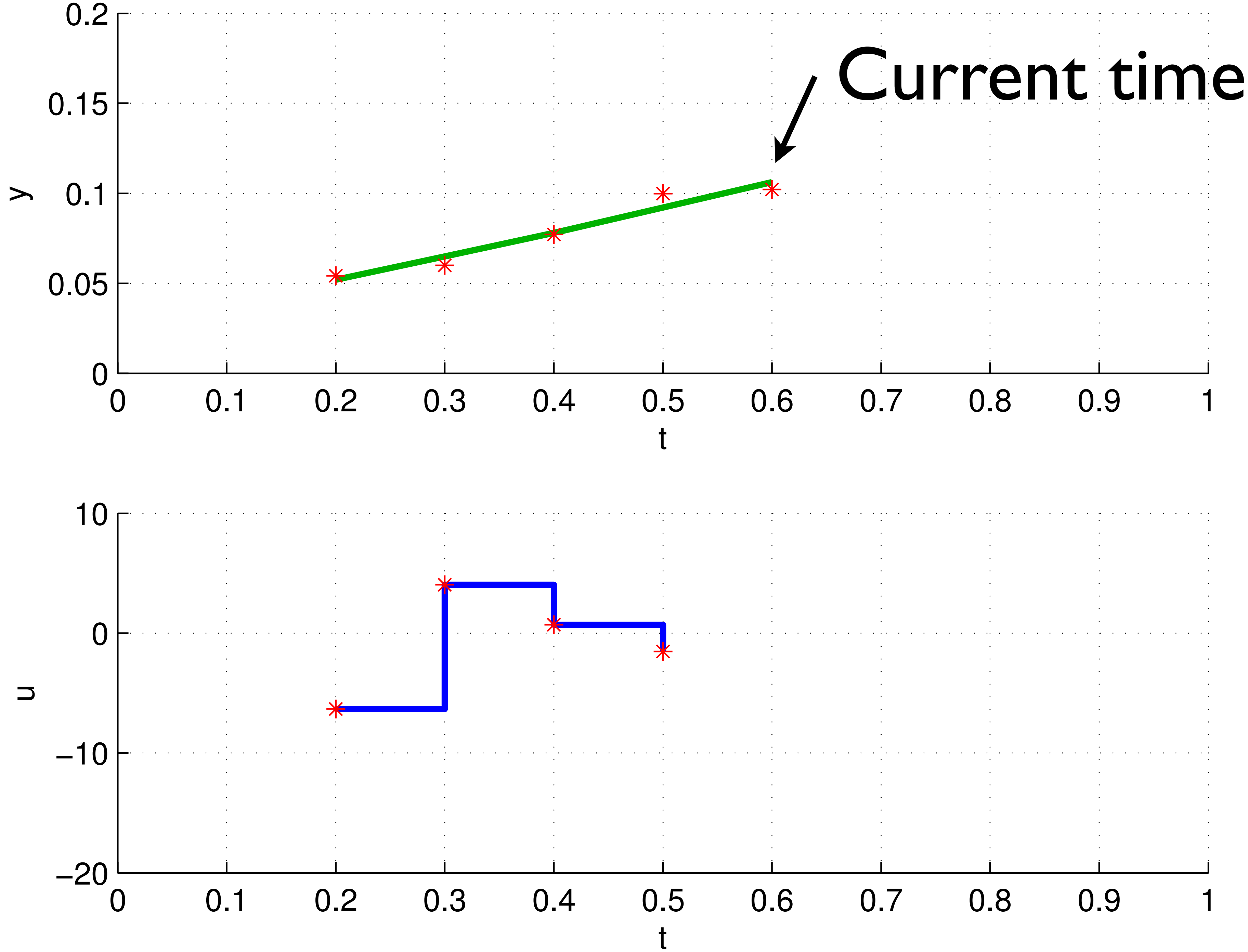
$$\min_{u,s} \quad \sum_{k=0}^{-E} \|y(s_k) - \bar{y}_k\|_Q^2 + \sum_{k=0}^{-E+1} \|u_k - \bar{u}_k\|_R^2 \quad \rightarrow$$ **mismatch** meas. model - meas.

$$\text{s.t.} \quad s_{k+1} = f(s_k, u_k) \quad \rightarrow$$ **model** of the system

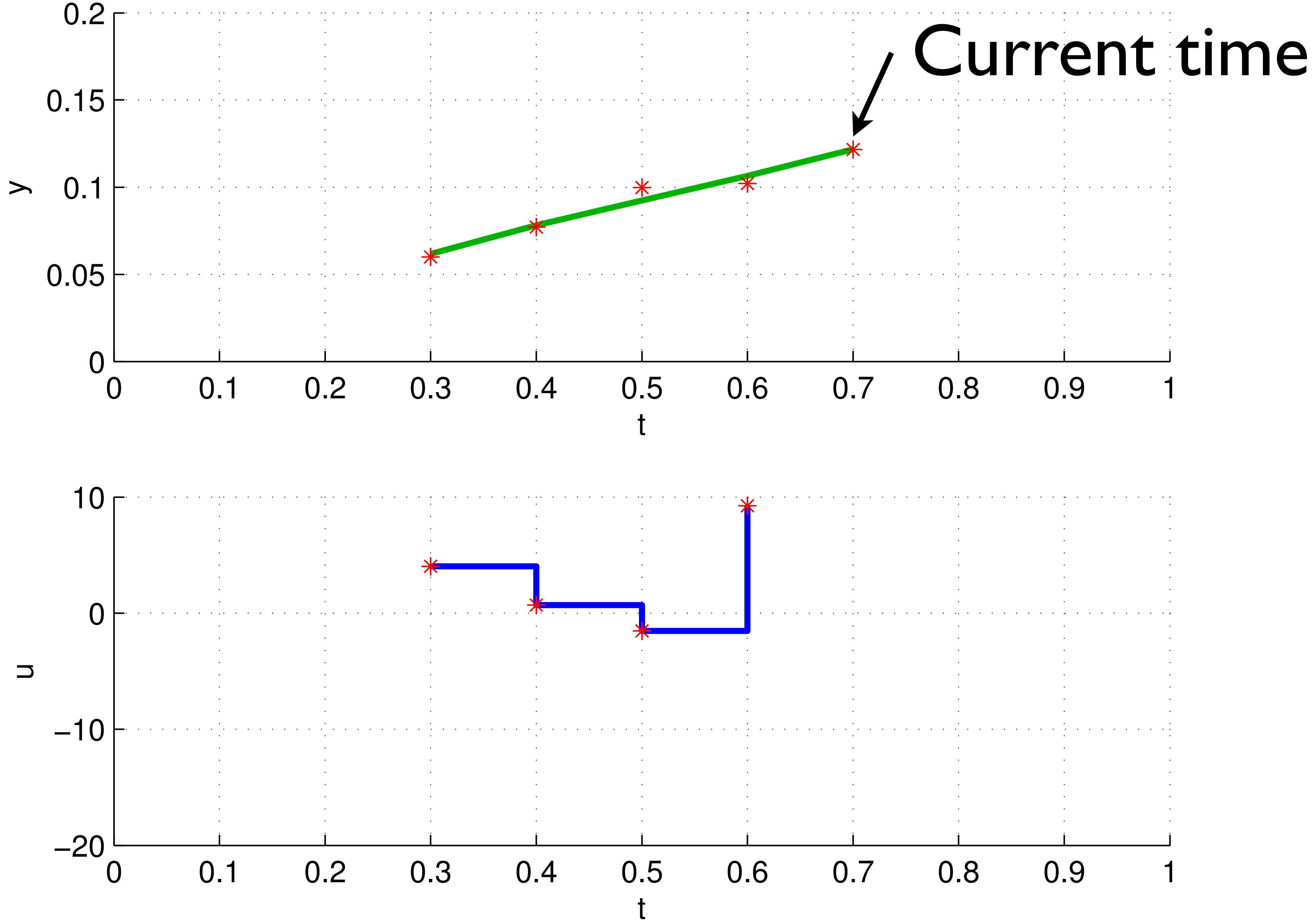
Nonlinear Moving Horizon Estimation



Nonlinear Moving Horizon Estimation



Nonlinear Moving Horizon Estimation



Optimal Control Problem

$$\min_{\substack{x_0,\ldots,x_N \\ u_0,\ldots,u_{N-1}}} \quad ||x_0 - x_{\mathrm{AC}}||^2_{S_{\mathrm{AC}}} + \sum_{k=0}^{N-1} ||h(x_k,u_k) - \tilde{y}_k||^2_{S_k} + ||h_N(x_N) - \tilde{y}_N||^2_{S_N}$$

$$\text{s.t.} \quad \begin{aligned}
& x_0 = \hat{x}_0 \\
& x_{k+1} = F(x_k, u_k, z_k) && \text{for } k = 0,\ldots,N-1 \\
& x_k^{\mathrm{lo}} \le x_k \le x_k^{\mathrm{up}} && \text{for } k = 0,\ldots,N \\
& u_k^{\mathrm{lo}} \le u_k \le u_k^{\mathrm{up}} && \text{for } k = 0,\ldots,N-1 \\
& r_k^{\mathrm{lo}} \le r_k(x_k,u_k) \le r_k^{\mathrm{up}} && \text{for } k = 0,\ldots,N-1 \\
& r_N^{\mathrm{lo}} \le r_N(x_n) \le r_N^{\mathrm{up}}
\end{aligned}$$

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s.t.

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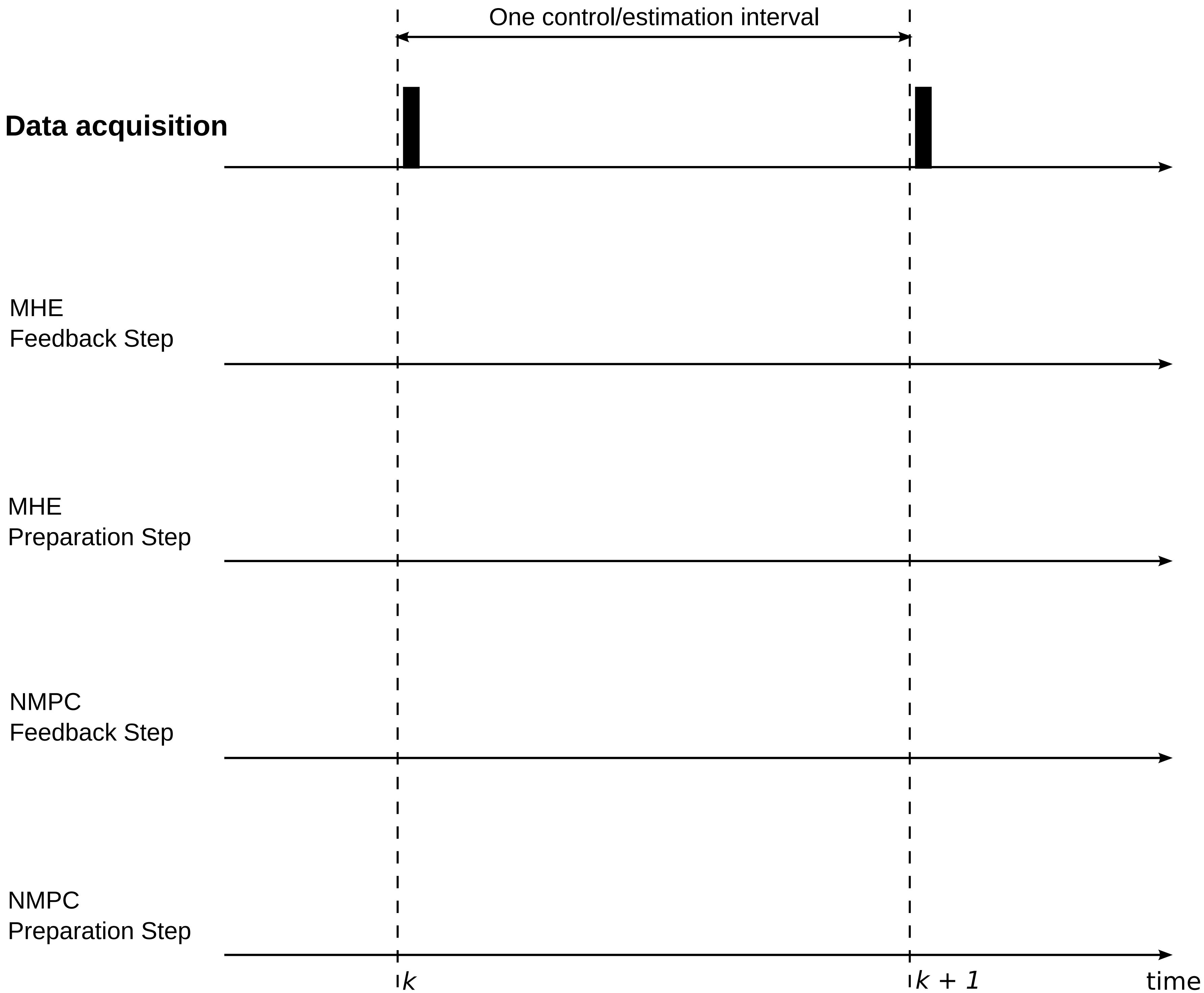
Solution methods

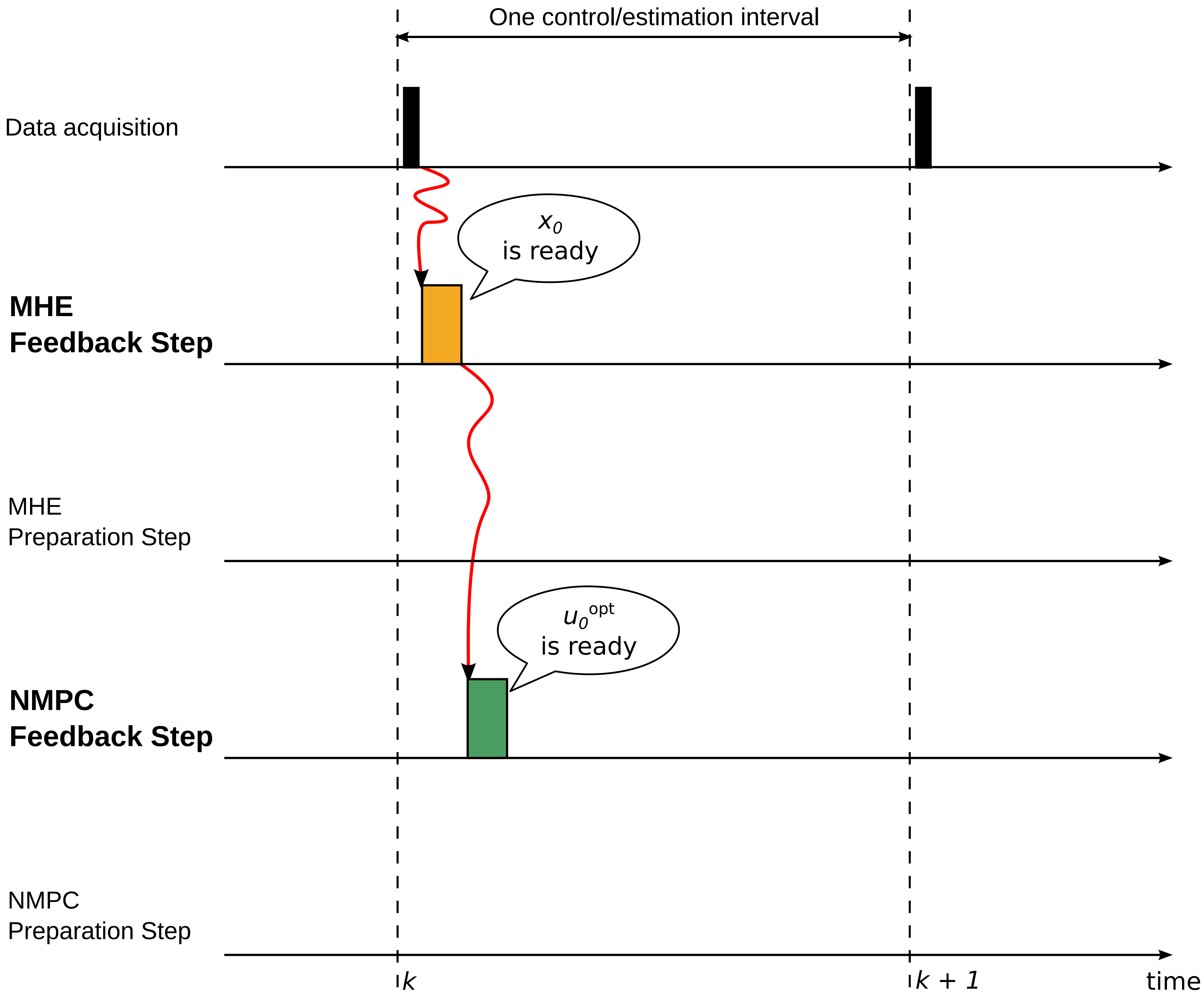
Real-time Iterations [Diehl 2002]

- Problem discretization - single/multiple shooting [Bock 1984]
- Least squares objective - employ Gauss-Newton method
- Perform only one SQP iteration per sampling time
- Optionally *condense* the OCP
- Division into preparation and feedback phase

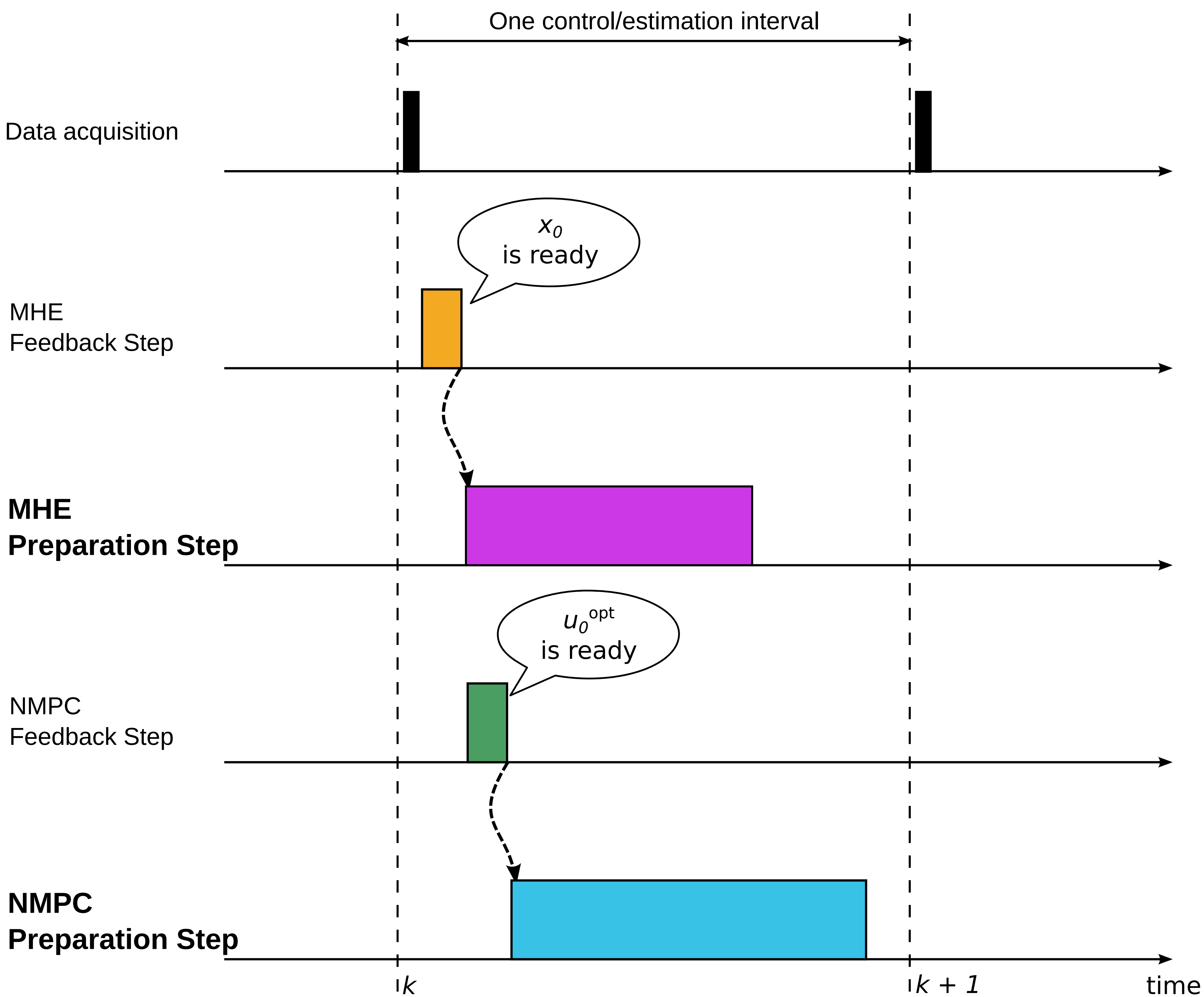
RTI Scheme 101(1)



RTI Scheme 101(2)



RTI Scheme 101(3)



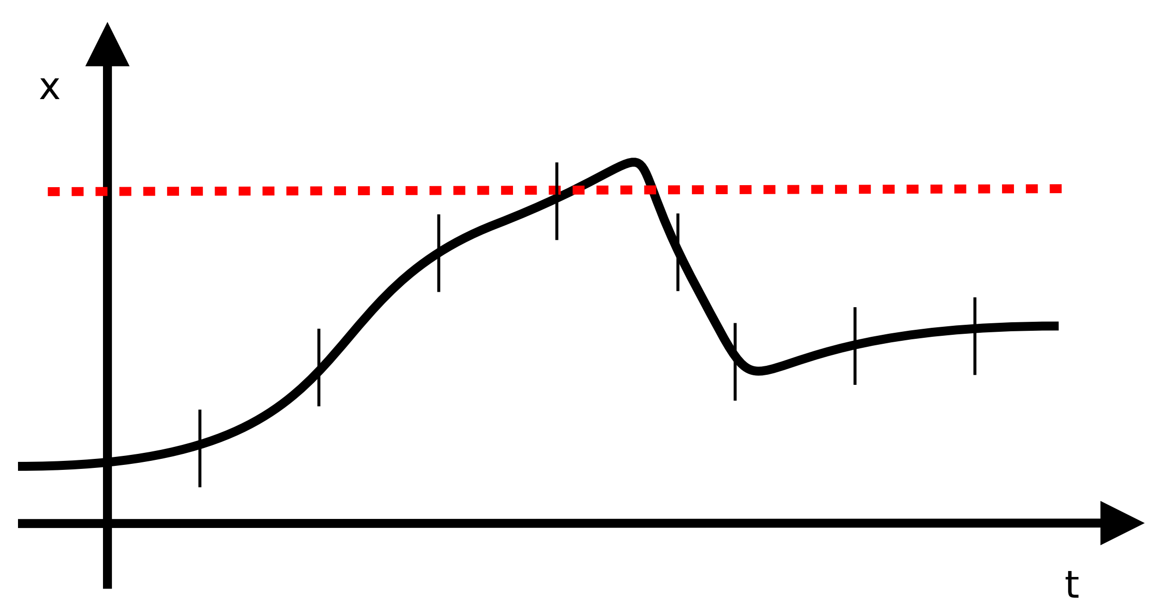
Continuous Output Integrators*

Motivation

- Multi-rate measurements - MHE
- Approximation of least-squares integrals

$$\int_0^T \|F(t,x,u)\|_2^2 \,\mathrm{d}t$$

- Checking constraints



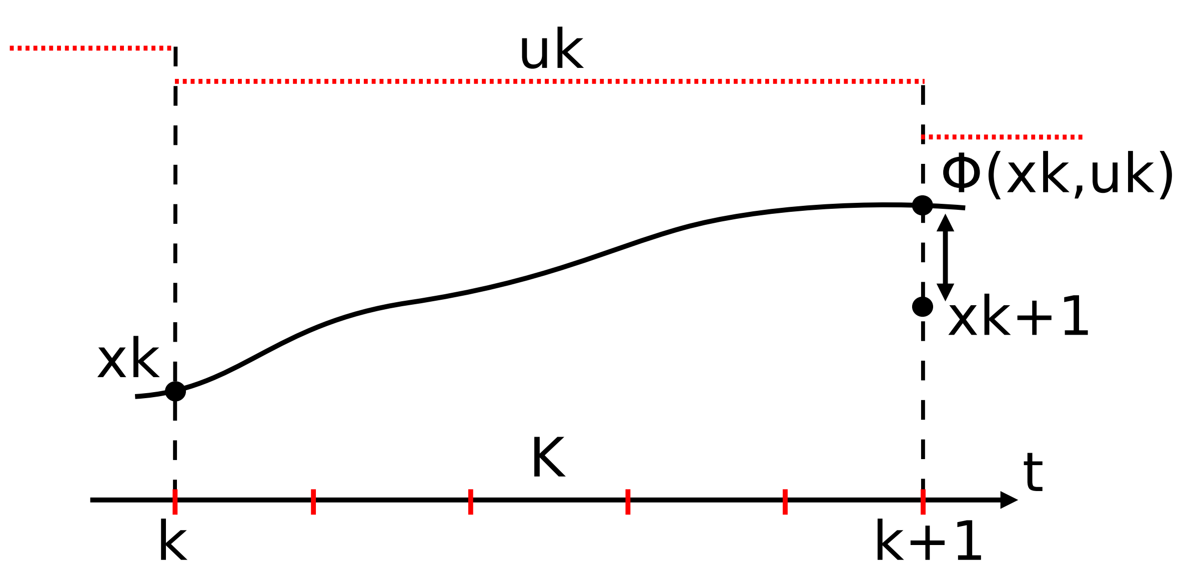
* Quirynen2012, Quirynen2013

Continuous Output Integrators

MHE Context

- Independent discretization grid
- Multi-rate measurements
- Very general measurement functions

$$y = \psi(t, \dot{x}(t), x(t), z(t))$$



ACADO toolkit [Houska 2009]

www.acadotoolkit.org

- Open source package (LGPL)
- Depends only on the standard C++ library
- Multi-platform: Linux, OS X, Windows
- MATLAB & Simulink Interfaces

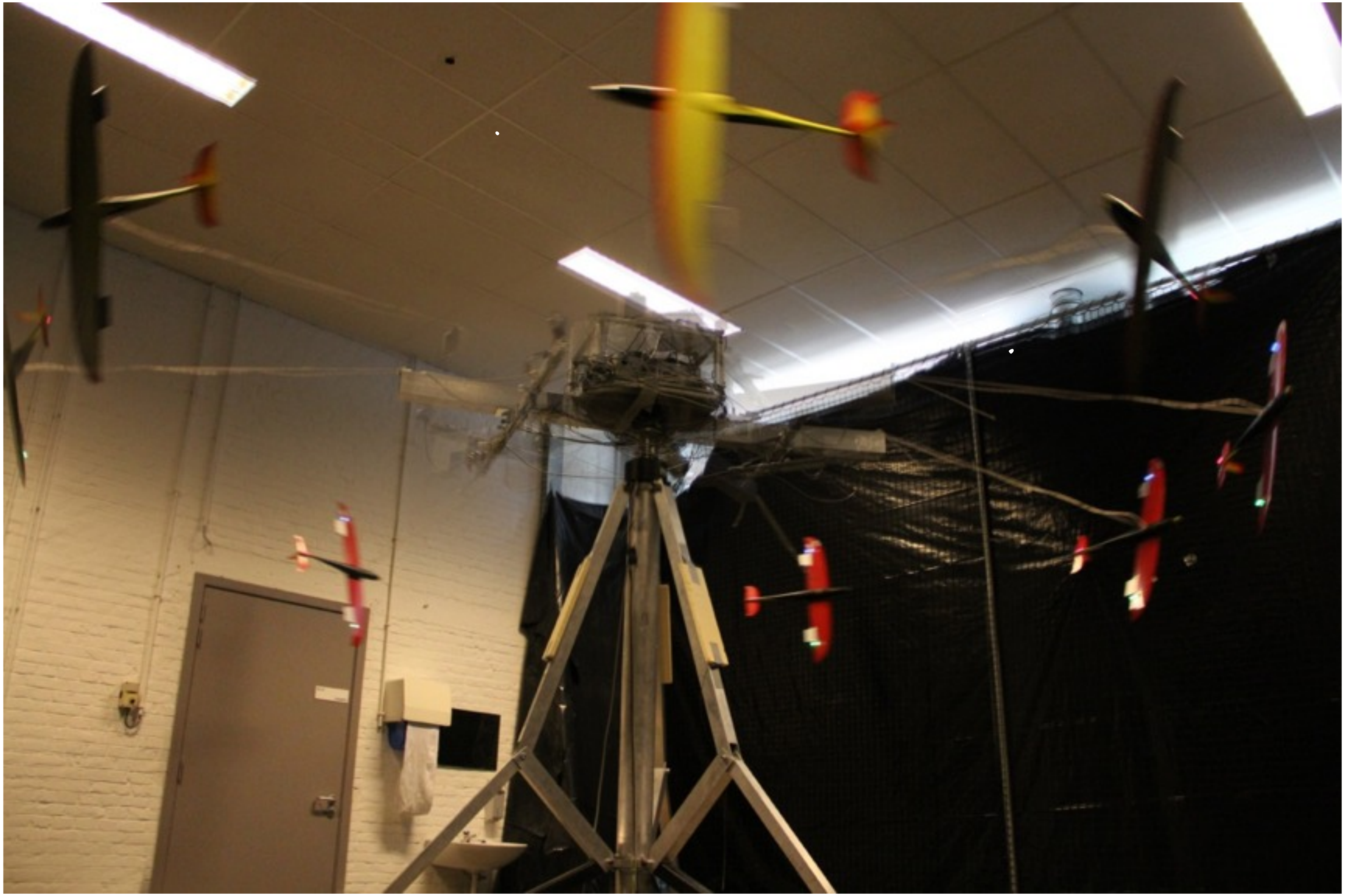
- Optimal control of dynamic systems
- State and parameter estimation
- Feedback control based on MPC/MHE
- Fast implementations for RT execution: ACADO Code

ACADO Code Generation Tool *

- Optimize the number of evaluations of the right-hand-side of ODE/DAE and its derivatives.
- Use tailored fixed-step Runge-Kutta integrators
- Avoid dynamic memory allocation
- Minimize branching in the exported code
- Export optimized linear algebra routines
- Interfaces to MATLAB & Simulink
- Interface to Python - `rawesome` by Greg Horn

* Houska2011, Ferreau2012, Quirynen2012, Vukov2012, Quirynen2013, Vukov2013

The Indoors Carousel



Carousel model *

Nonlinear dynamics based DAE

Translational:
$$\begin{bmatrix} m & 0 & 0 & x \\ 0 & m & 0 & y \\ 0 & 0 & m & z \\ x & y & z & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \lambda \end{bmatrix} = \begin{bmatrix} F_x + m\left(\dot{\delta}^2 r_A + \dot{\delta}^2 x + 2\dot{\delta}\dot{y} + \ddot{\delta} y\right) \\ F_y + m\left(y\dot{\delta}^2 - 2\dot{x}\dot{\delta} - \ddot{\delta}(rA + x)\right) \\ F_z - gm \\ -\dot{x}^2 - \dot{y}^2 - \dot{z}^2 \end{bmatrix}$$

Rotational:
$$\dot{R} = R\omega_\times - R^T \begin{bmatrix} 0 \\ 0 \\ \dot{\delta} \end{bmatrix}, \qquad J\dot{\omega} = T - \omega \times J\omega, \qquad R = \begin{bmatrix} \vec{E}_x & \vec{E}_y & \vec{E}_z \end{bmatrix}$$

Aero. coefficients:
$$\vec{v} = \begin{bmatrix} \dot{x} - \dot{\delta} y \\ \dot{y} + \dot{\delta}(r_\mathrm{A} + x) \\ \dot{z} \end{bmatrix} - \vec{w}(x, y, z, \delta, t), \qquad \alpha = -\frac{\vec{E}_z^T \vec{v}}{\vec{E}_x^T \vec{v}}, \qquad \beta = \frac{\vec{E}_y^T \vec{v}}{\vec{E}_x^T \vec{v}}$$

Aero. forces/torques:
$$\vec{F}_\mathrm{A} = \frac{1}{2}\rho A \|\vec{v}\| (C_\mathrm{L} \vec{v} \times \vec{E}_y - C_\mathrm{D} \vec{v}), \quad \vec{T}_\mathrm{A} = \frac{1}{2}\rho A \|\vec{v}\|^2 \begin{bmatrix} C_\mathrm{R} \\ C_\mathrm{P} \\ C_\mathrm{Y} \end{bmatrix}$$

* Gros2012, AWE Book

MHE Setup

- Measurements:
  - IMU: gyros, accelerometers (500 Hz)
  - Stereo-vision system (12.5 Hz)
  - Encoder (1 kHz)
  - Control signals (50 Hz)

MHE Setup

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  - IMU: g Averaged s (500 Hz)
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  - Encoder (1 kHz)
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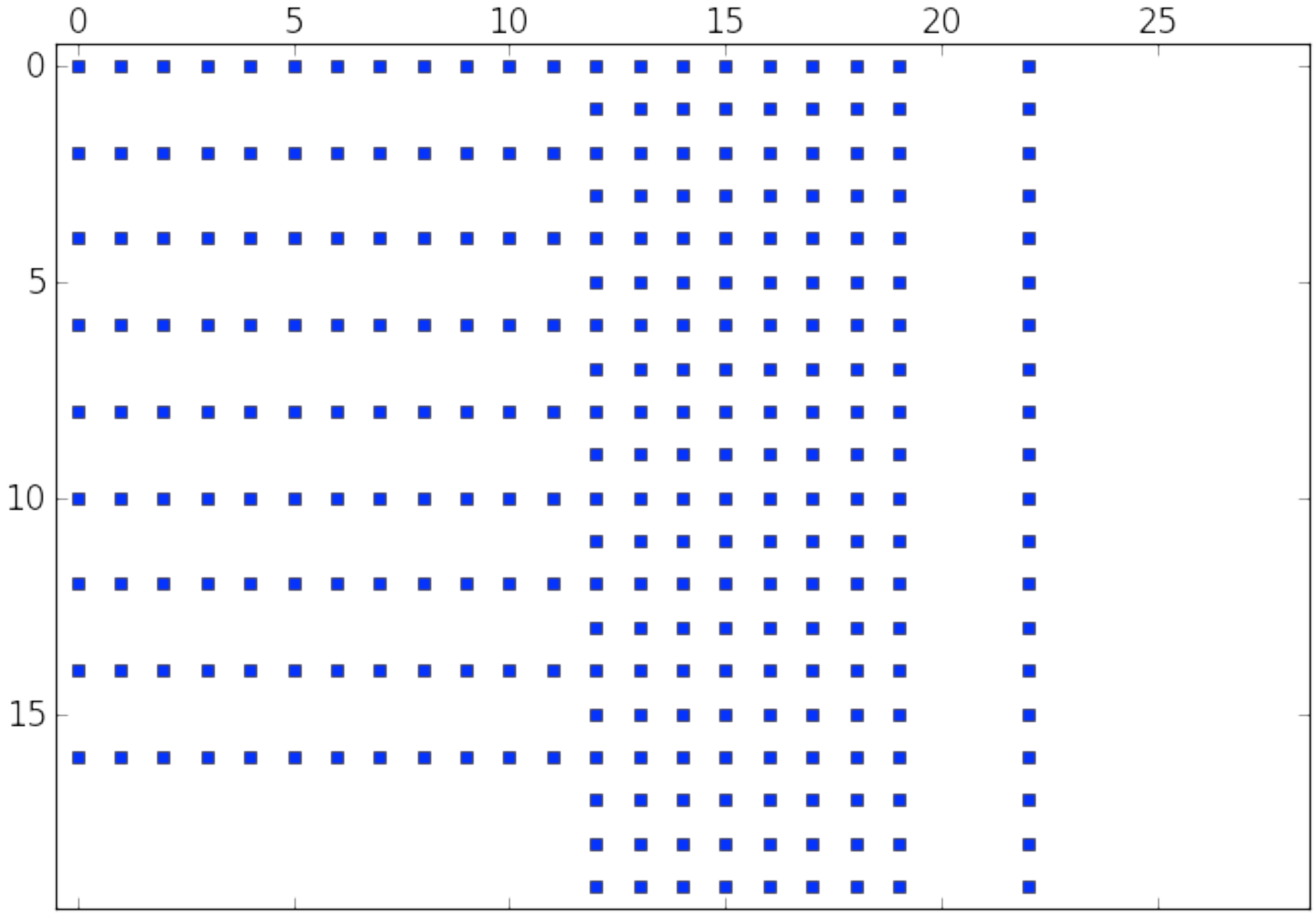
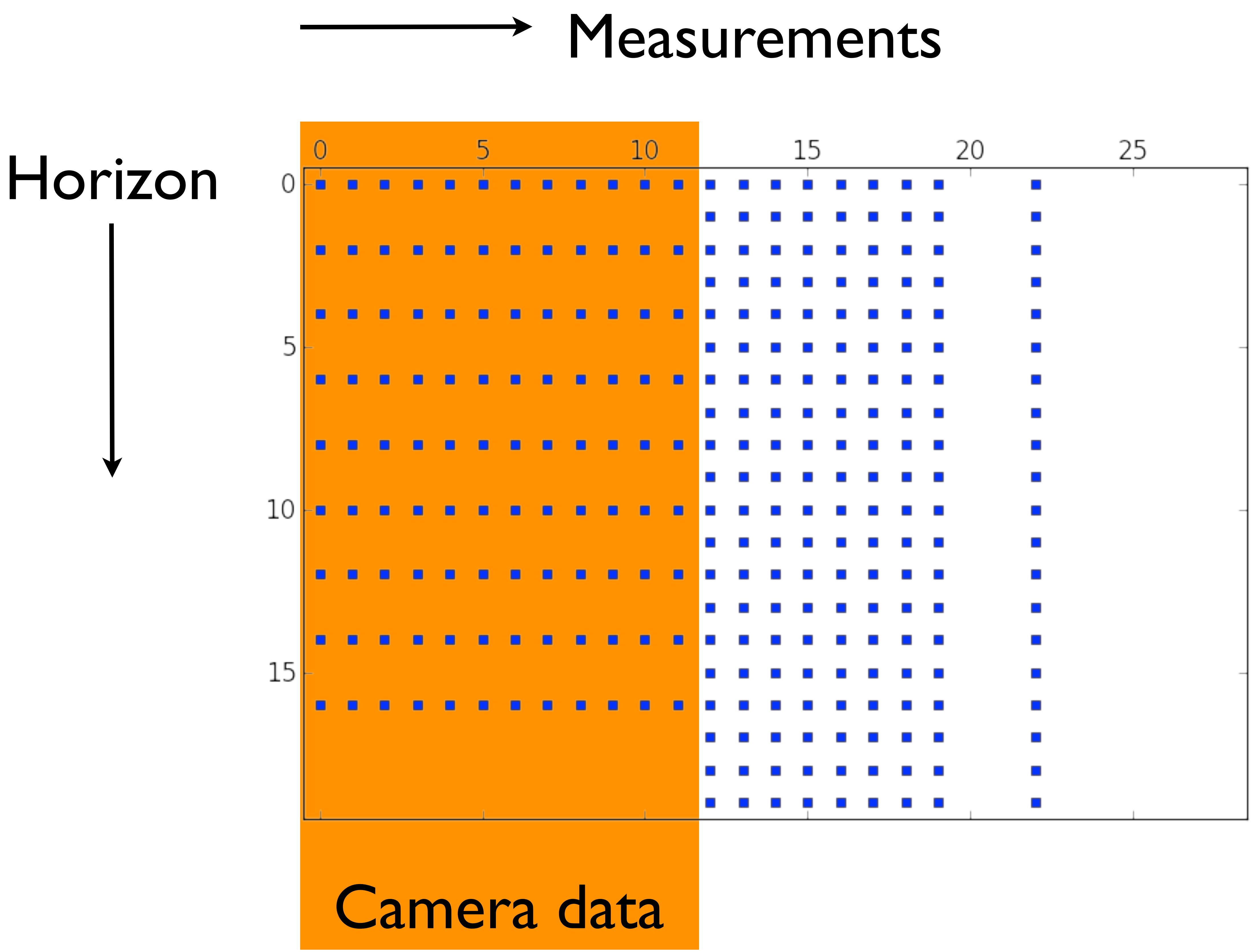
Measurements

Horizon

Measurements

Horizon





Measurements
Horizon
0
5
10
15
20
25
Camera data

MHE Setup

- Optimal Control Problem formulation:
  - 27 differential states
  - 1 algebraic state
  - 4 controls
  - Discretization time 0.04 s (25 Hz)
  - 20 control intervals

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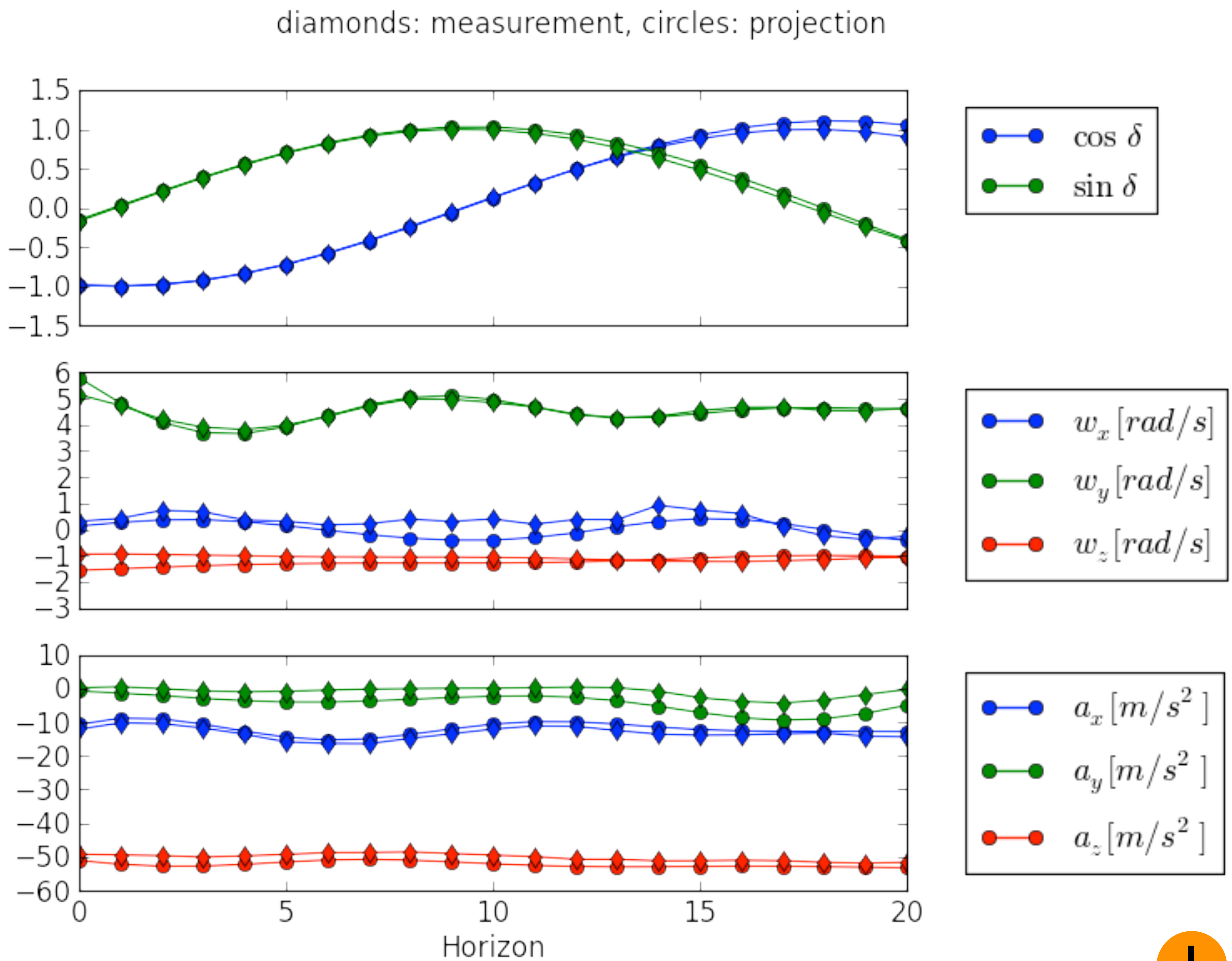
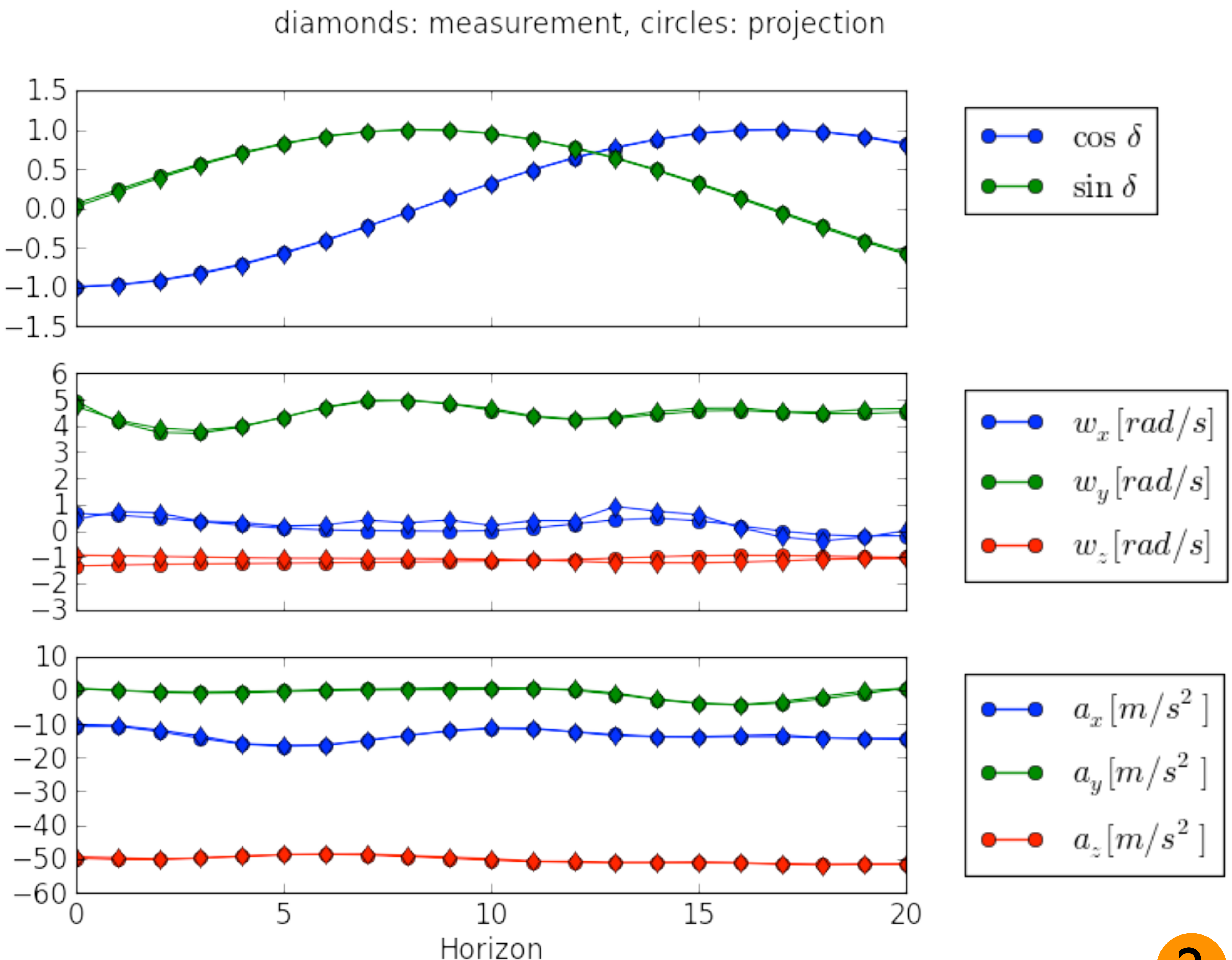
MHE - Results

- We verified the MHE solver on experimental data
- The solver is real-time feasible @ 25 Hz



diamonds: measurement, circles: projection
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
cos δ
sin δ
6
5
4
3
2
1
0
−1
−2
−3
$w_x\ [rad/s]$
$w_y\ [rad/s]$
$w_z\ [rad/s]$
10
0
−10
−20
−30
−40
−50
−60
$a_x\ [m/s^2]$
$a_y\ [m/s^2]$
$a_z\ [m/s^2]$
0
5
10
15
20
Horizon



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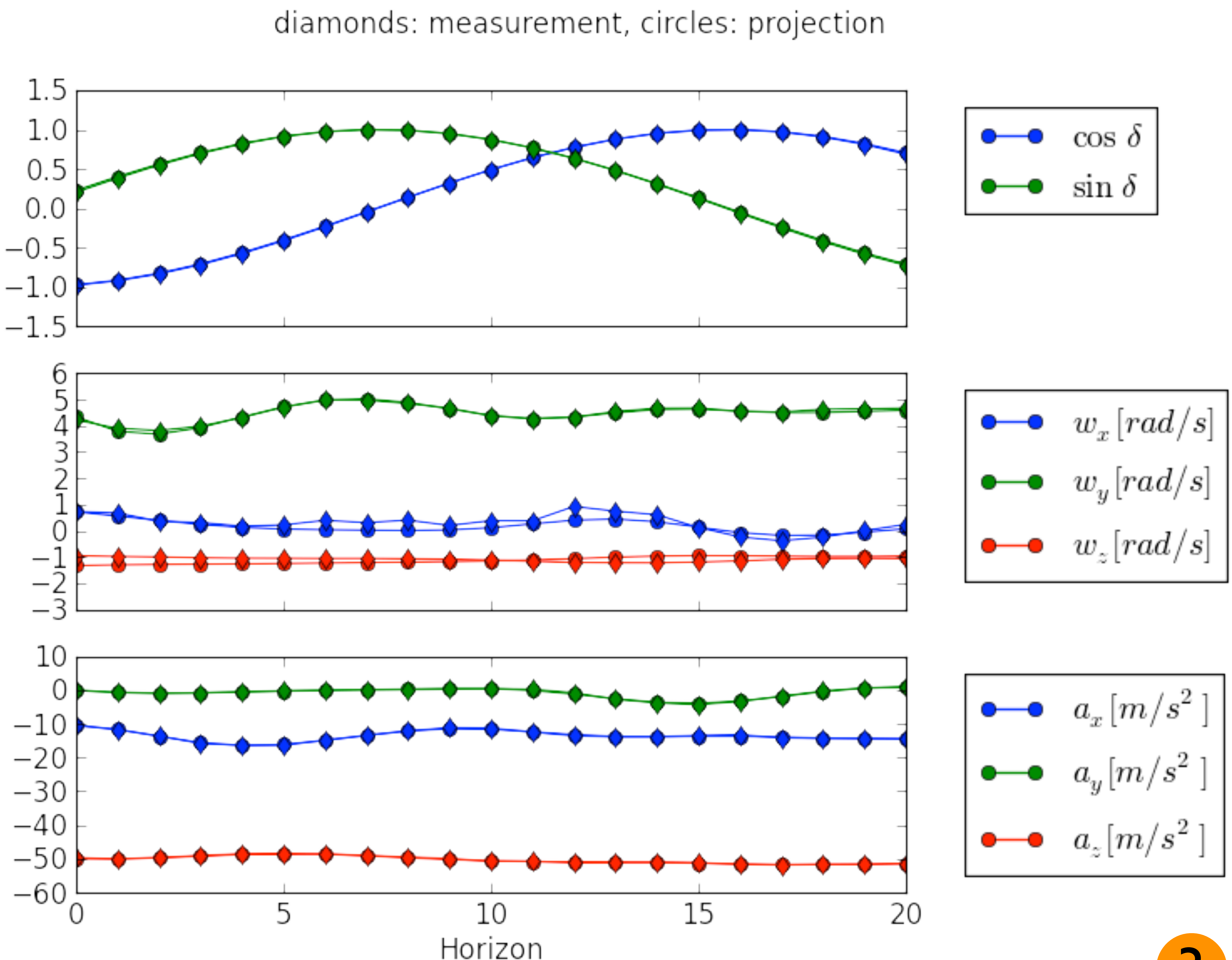
−2
−3
$w_x\,[rad/s]$
$w_y\,[rad/s]$
$w_z\,[rad/s]$
10
0
−10
−20
−30
−40
−50
−60
0
5
10
15
20
Horizon
$a_x\,[m/s^2]$
$a_y\,[m/s^2]$
$a_z\,[m/s^2]$

2


diamonds: measurement, circles: projection
1.5
1.0
0.5
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−10
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−30
−40
−50
−60
0
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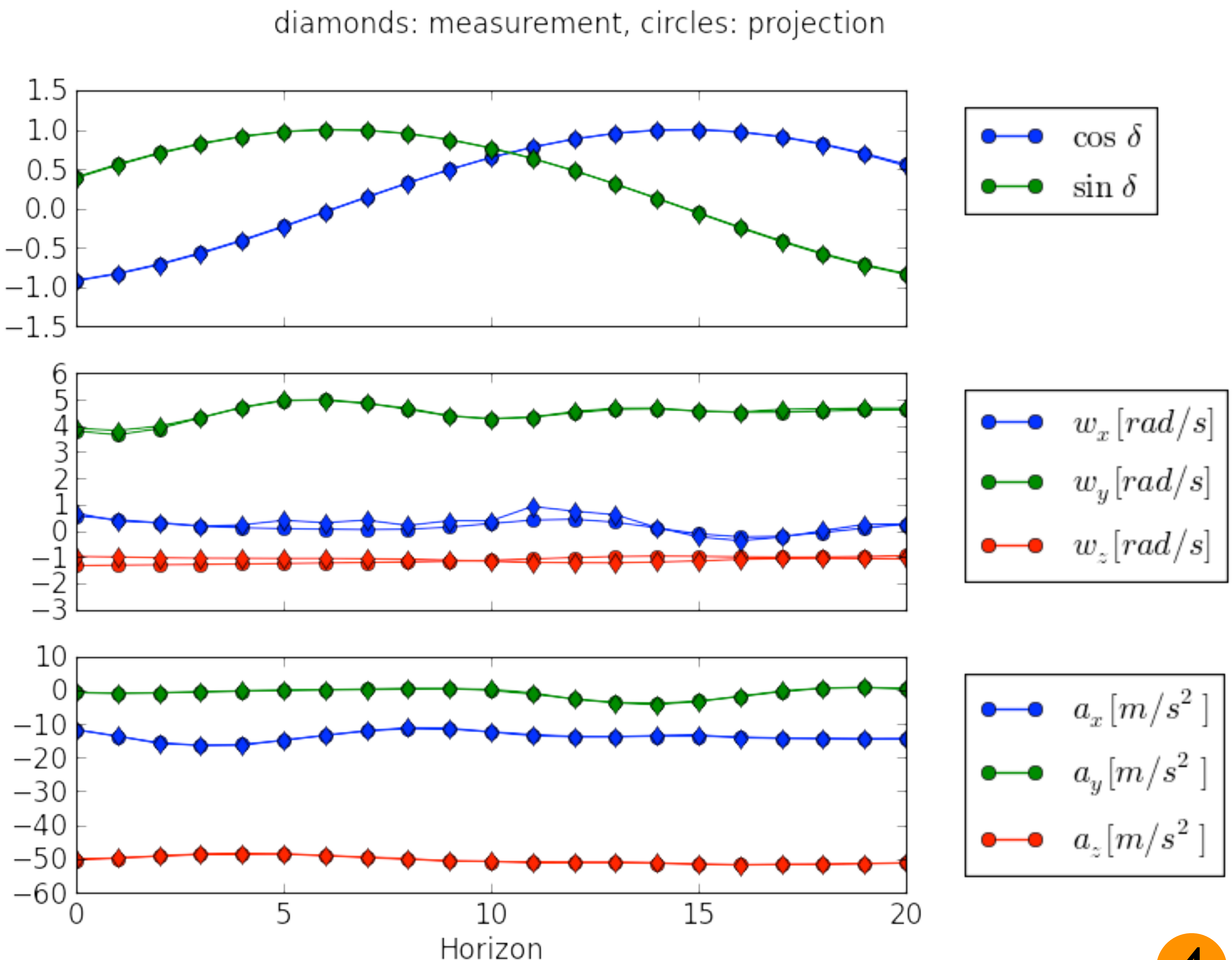
Horizon
$a_x\ [m/s^2]$
$a_y\ [m/s^2]$
$a_z\ [m/s^2]$

3



diamonds: measurement, circles: projection
1.5
1.0
0.5
0.0
−0.5
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−1.5
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$w_x\,[rad/s]$
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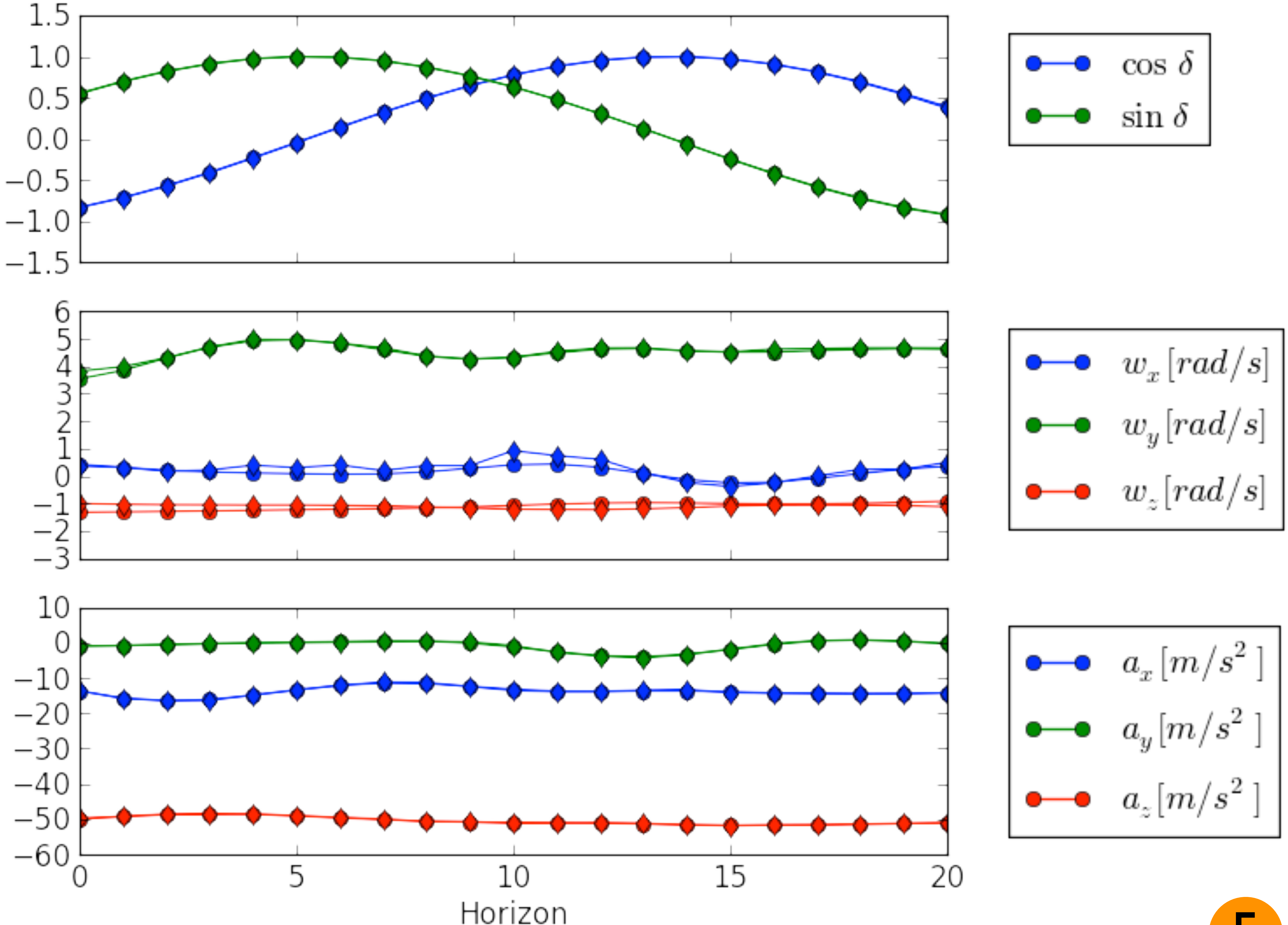
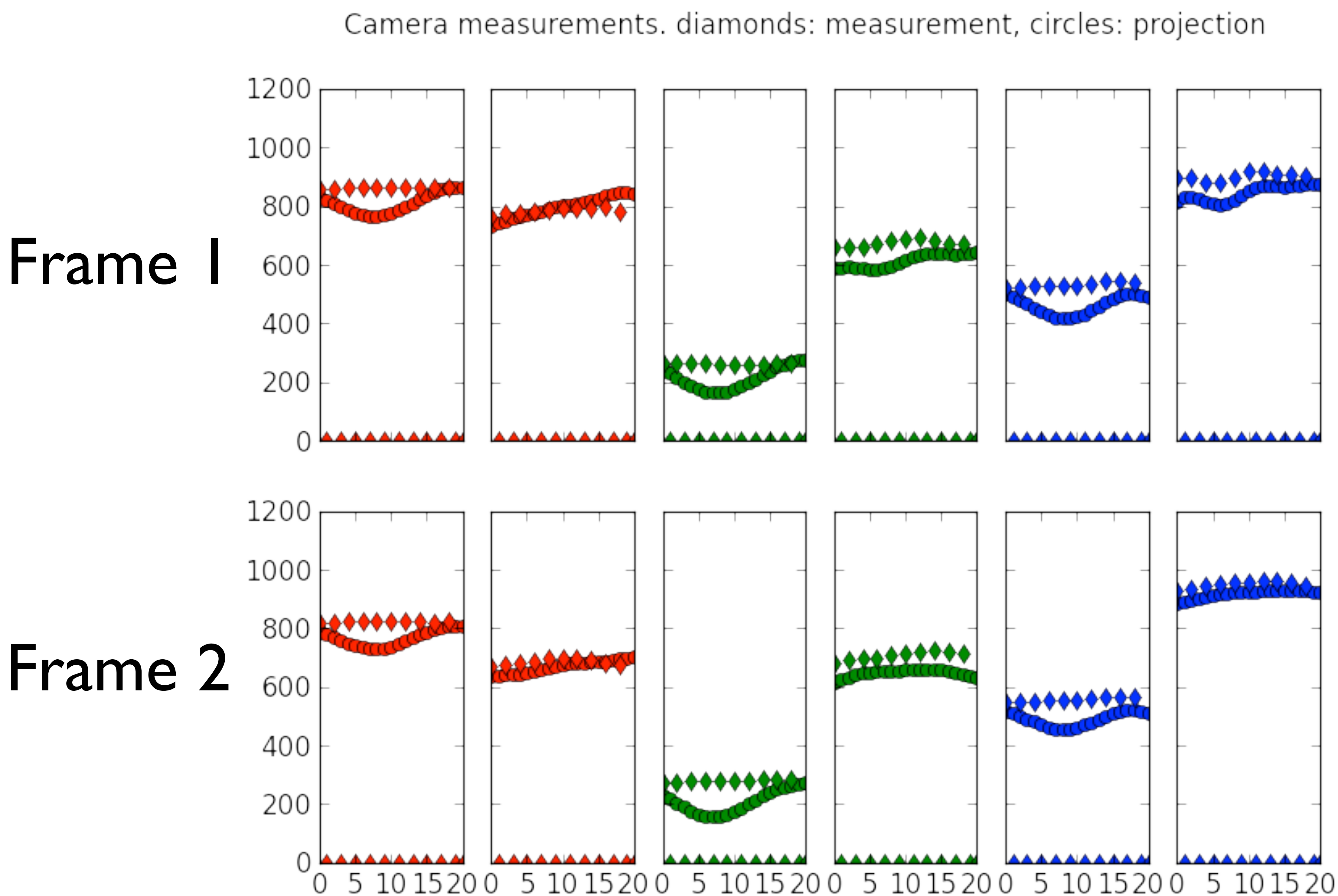
−30
−40
−50
−60
0
5
10
15
20
Horizon
$a_x\,[m/s^2]$
$a_y\,[m/s^2]$
$a_z\,[m/s^2]$

4

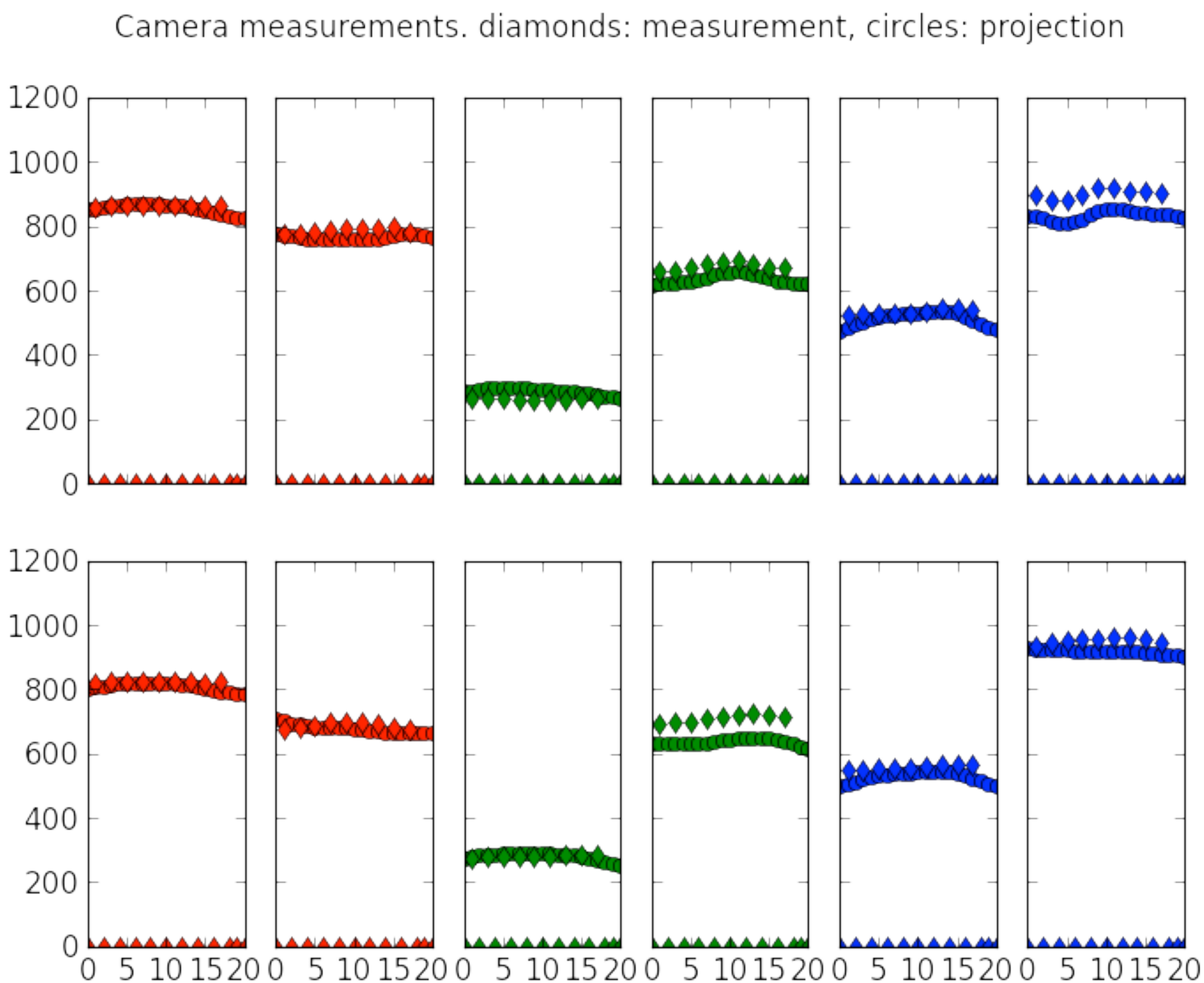
diamonds: measurement, circles: projection



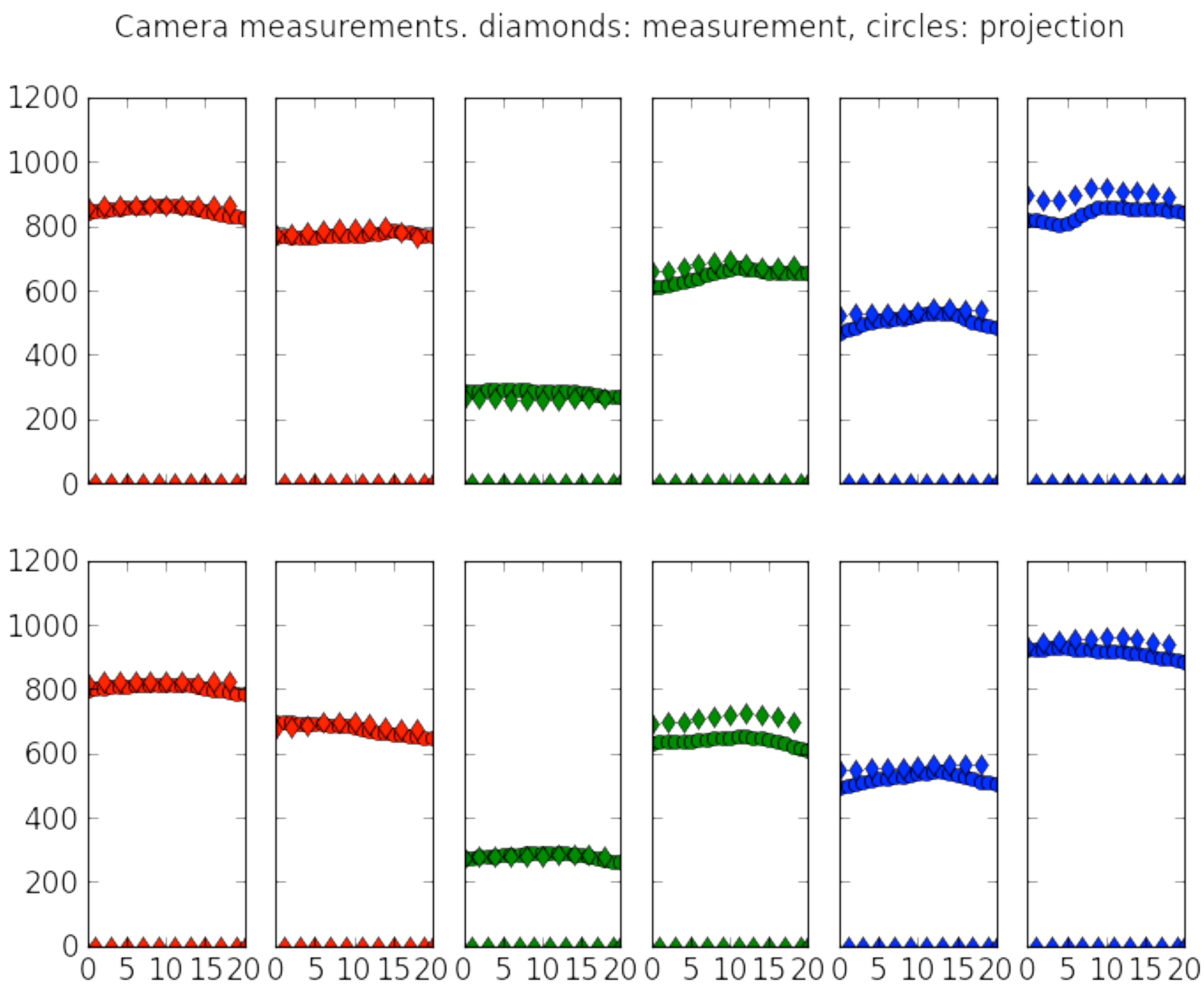
5


Camera measurements. diamonds: measurement, circles: projection
Frame 1
Frame 2
1200
1000
800
600
400
200
0
0 5 10 15 20

I

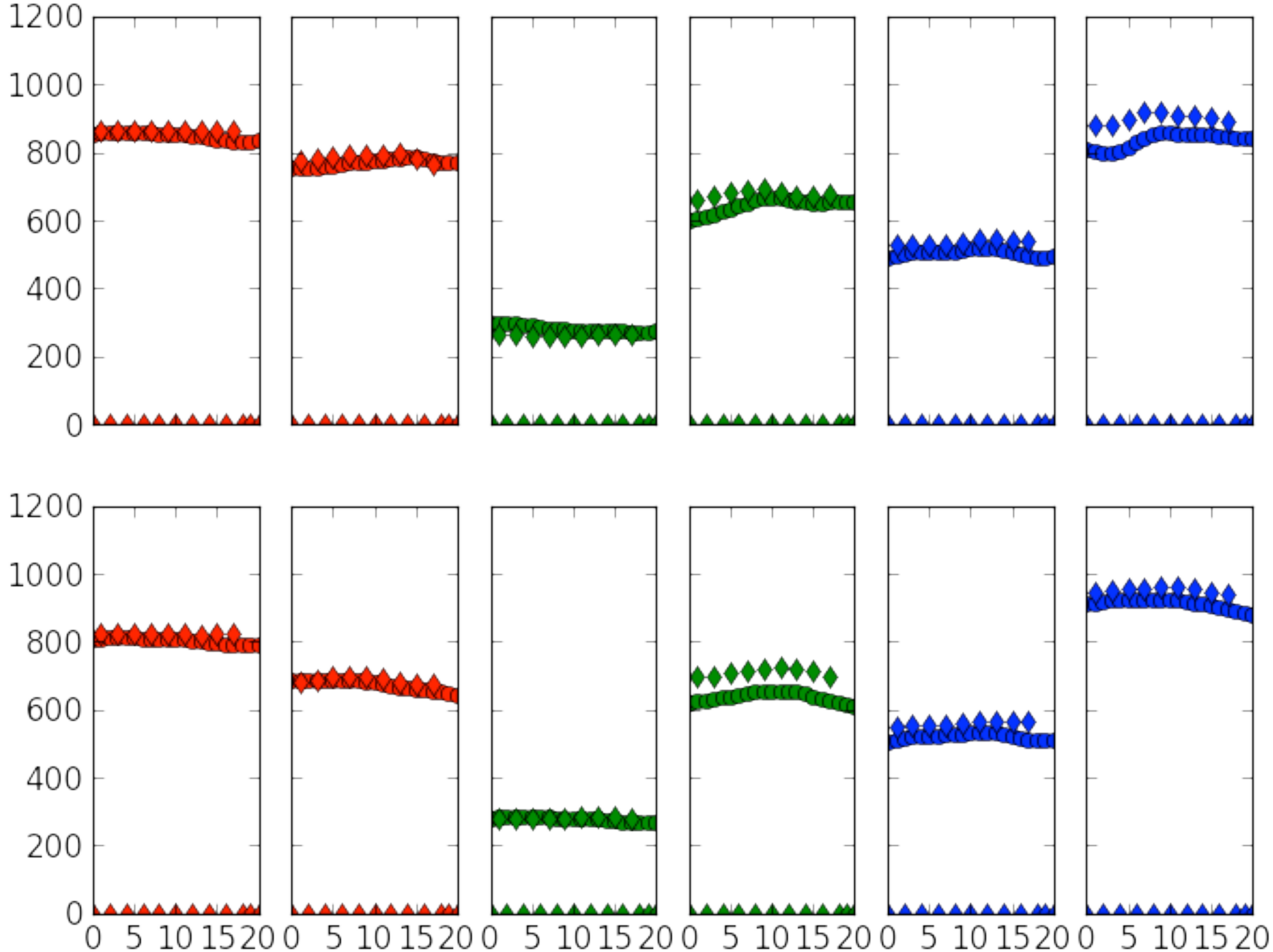

Camera measurements. diamonds: measurement, circles: projection
1200
1000
800
600
400
200
0
0 5 10 15 20

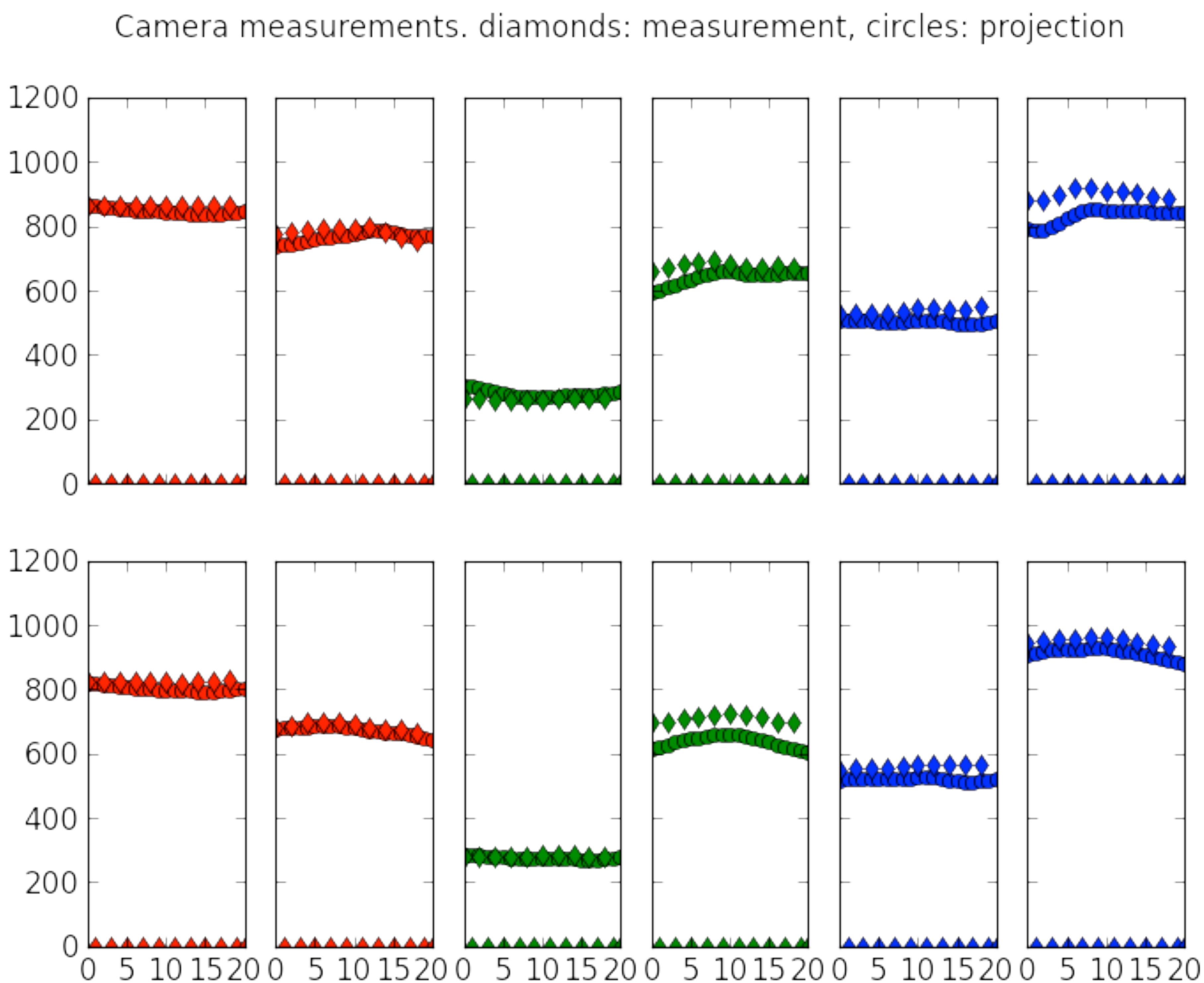
2


Camera measurements. diamonds: measurement, circles: projection
1200
1000
800
600
400
200
0
0 5 10 15 20

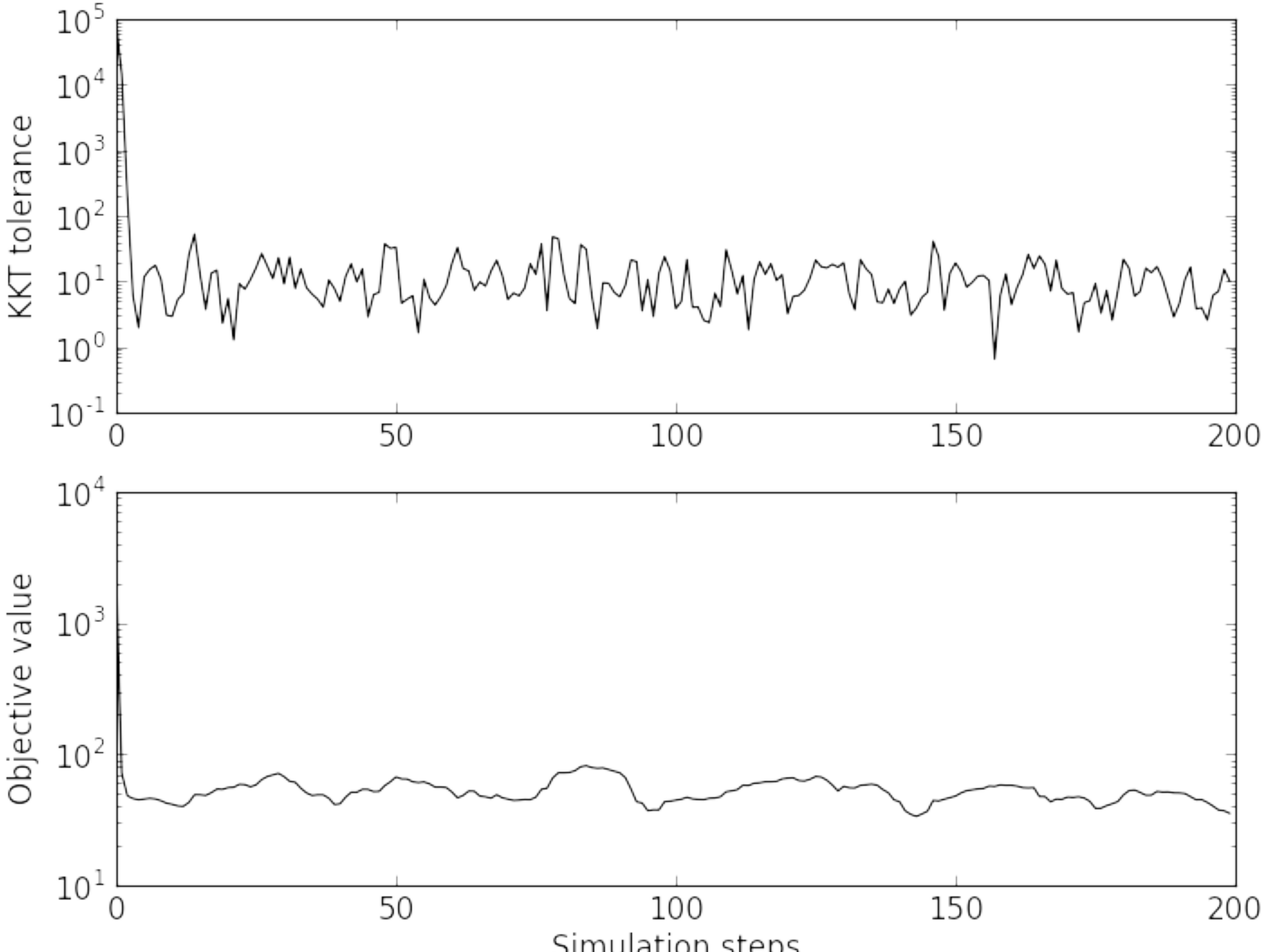
3

Camera measurements. diamonds: measurement, circles: projection

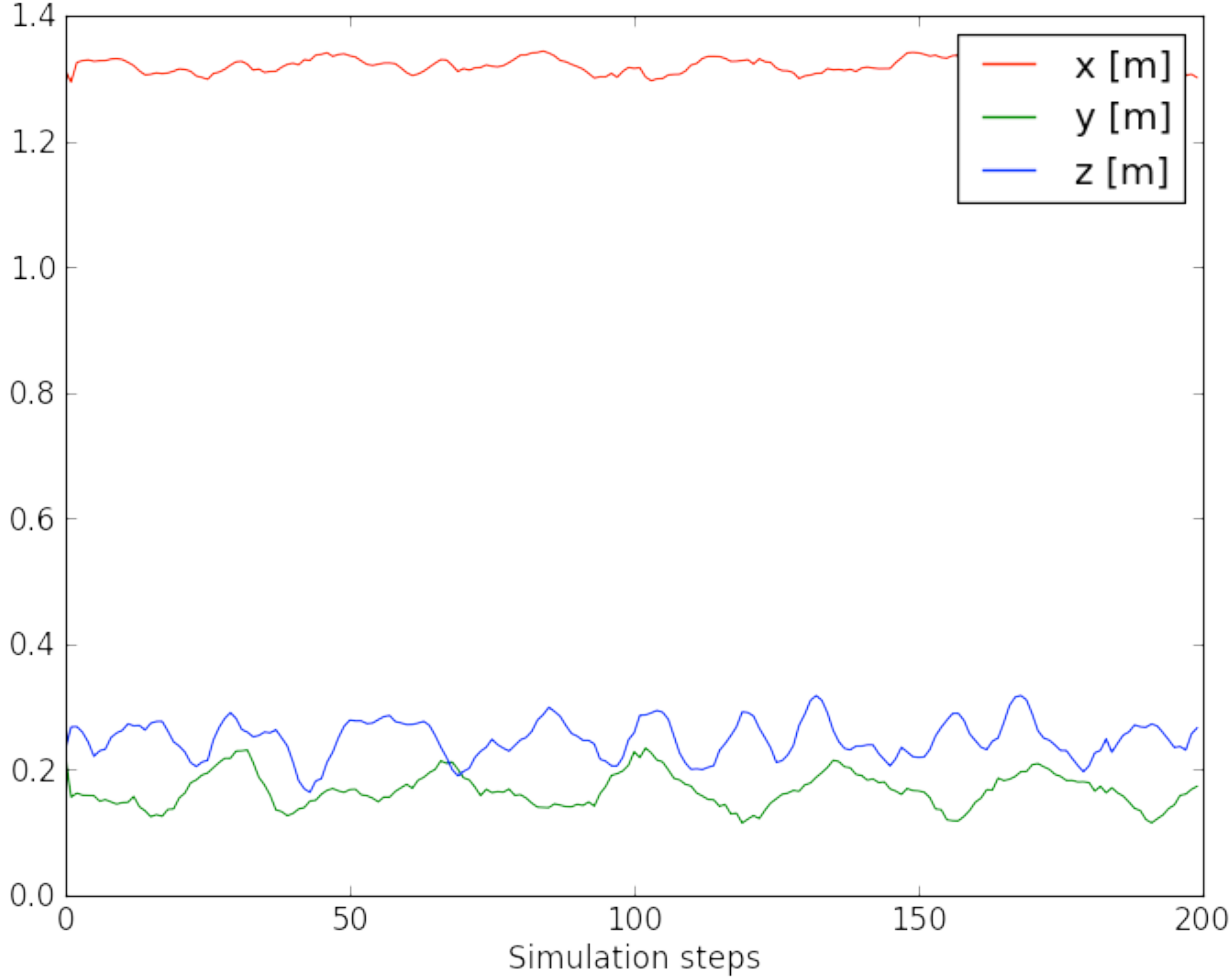



Camera measurements. diamonds: measurement, circles: projection
1200
1000
800
600
400
200
0
0 5 10 15 20

MHE performance



Position estimates



North

East

Down

Thank you very much for your attention!

Questions?